Explicit Density Learning

INTELLIGENT SYSTEMS FOR PATTERN RECOGNITION (ISPR)

DAVIDE BACCIU – DIPARTIMENTO DI INFORMATICA - UNIVERSITA’ DI PISA

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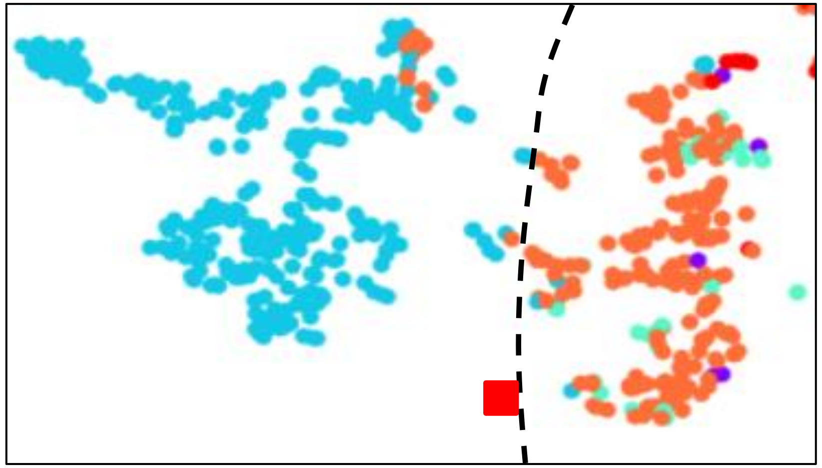
Lecture Outline

- Introduction to the Generative DL module
  - Motivations and taxonomy
- Explicit generative learning (Part I of III)
  - Learning distributions with fully visible information (RNN)
  - Learning distributions with latent information (VAE)
- VAE Application Examples

Generative DL Module

Why Generative?

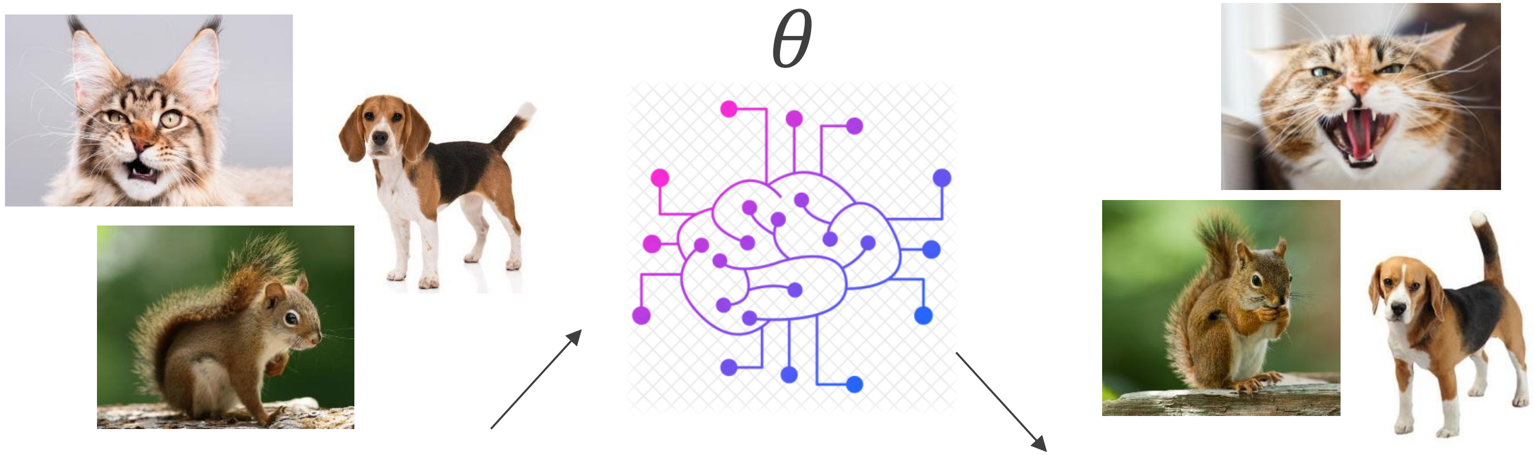
- Focusing too much on discrimination rather than on characterizing data can cause issues
  - Reduced interpretability
  - Adversarial examples



- Generative models (try to) characterize data distribution
  - Understand the data $\Longrightarrow$ Understand the world
  - Understand data variances $\Longrightarrow$ Learn to steer them
  - Understand normality $\Longrightarrow$ Detect anomalies

Approaching the Problem from a DL Perspective

Given training data, learn a (deep) neural network that can generate new samples from (an approximation of) the data distribution



Training data $\sim P(x)$

Generated data $\sim P_\theta(x)$

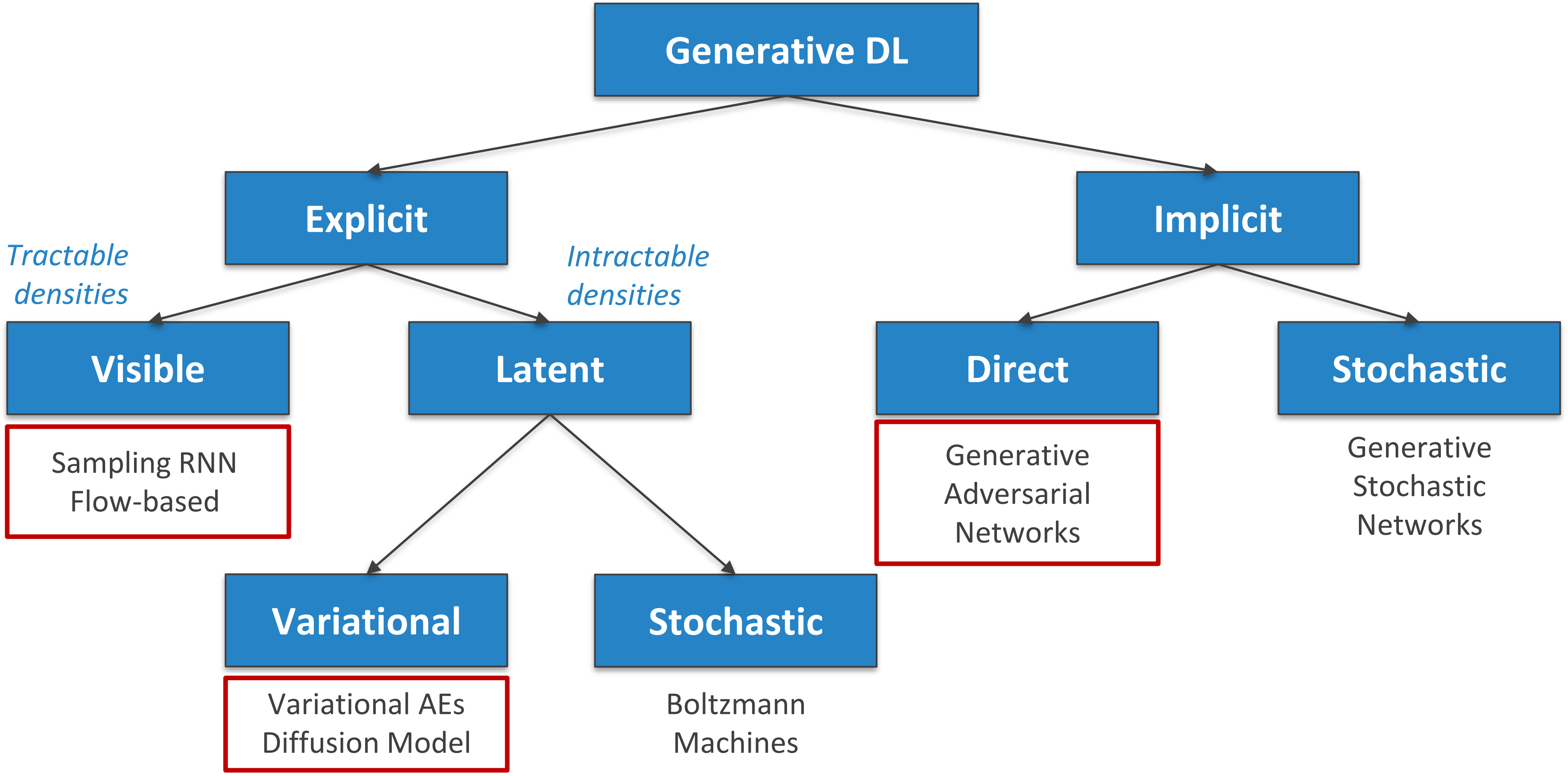
Approaching the Problem from a DL Perspective

Given training data, learn a (deep) neural network that can generate new samples from (an approximation of) the data distribution

Two approaches

- Explicit $\Longrightarrow$ Learn a model density $P_\theta(x)$
- Implicit $\Longrightarrow$ Learn a process that samples data from $P_\theta(x) \approx P(x)$

A Taxonomy



Adapted from I. Goodfellow, Tutorial on Generative Adversarial Networks, 2017

Density Learning with Full Observability

Learning with Fully Visible Information

If all information is fully visible the joint distribution can be computed from the chain rule factorization

Bayesian Networks →

$$P(\boldsymbol{x}) = \prod_{i}^{N} P(x_i | x_1, \ldots, x_{i-1})$$

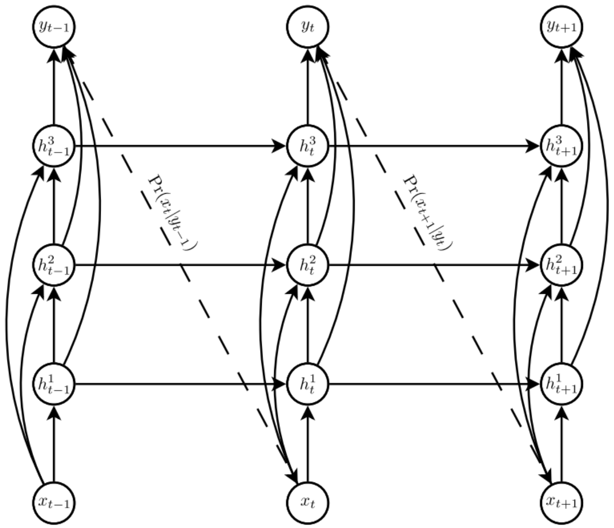
Probability of a pixel having a certain intensity value, given the known intensity of its predecessor

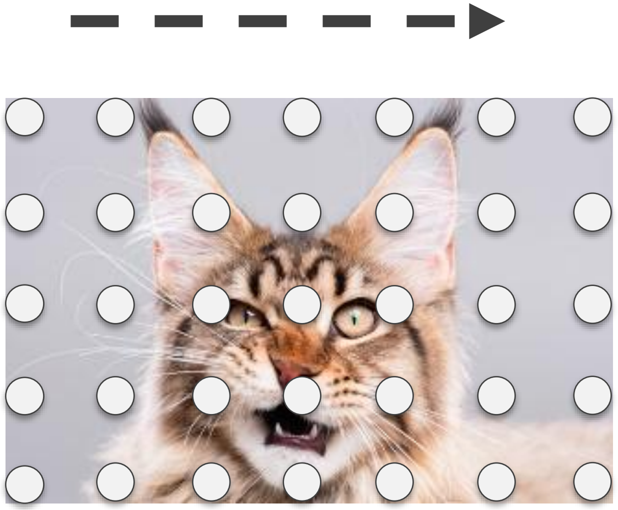
Need to be able to define a sensible ordering for the chain rule

Conditional distribution difficult to compute

Approximating the Conditional Probability

If all information is fully visible the joint distribution can be computed from the chain rule factorization





Scan the image according to a schedule and encode the dependency from previous pixels in the states of an RNN

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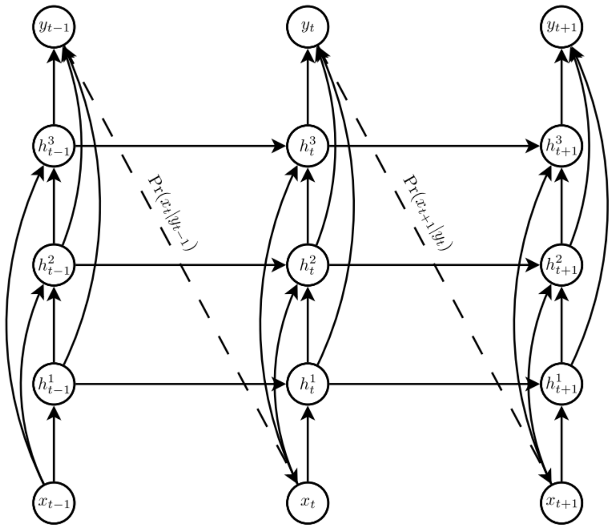
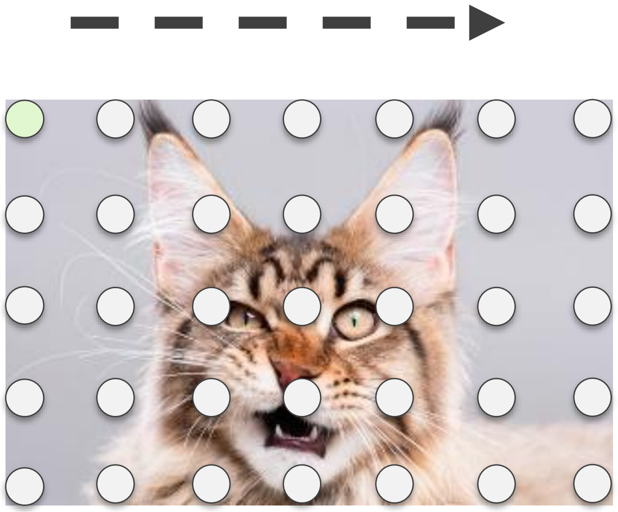
Approximating the Conditional Probability

If all information is fully visible the joint distribution can be computed from the chain rule factorization





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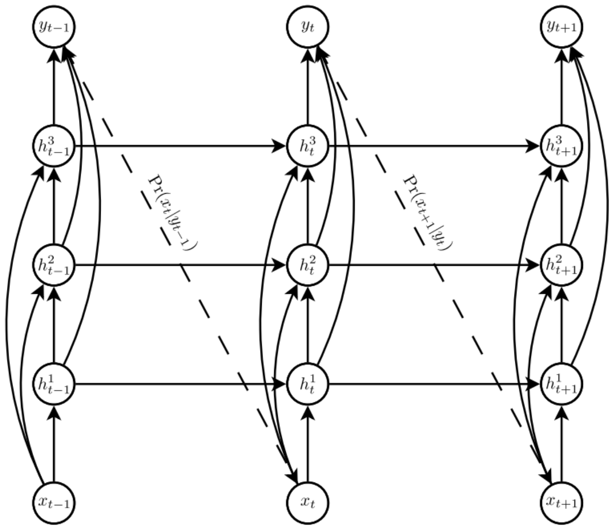
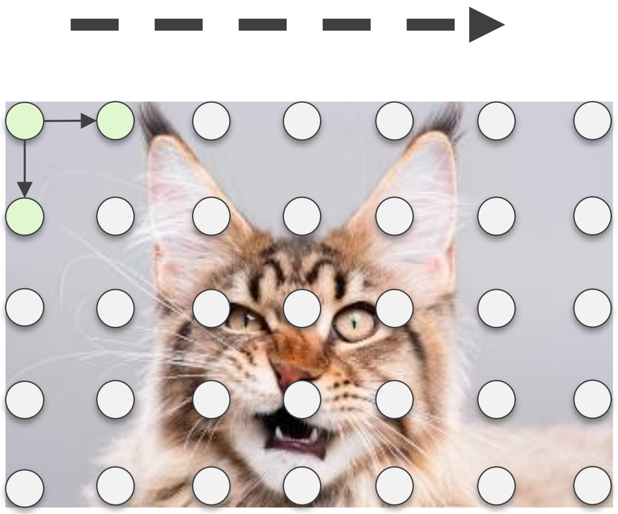
Approximating the Conditional Probability

If all information is fully visible the joint distribution can be computed from the chain rule factorization





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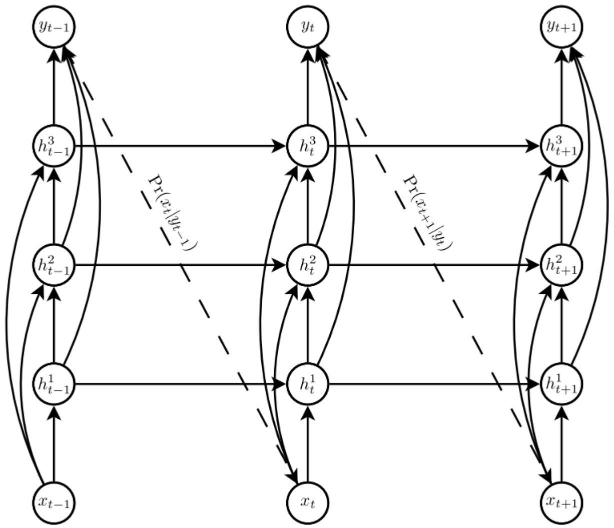

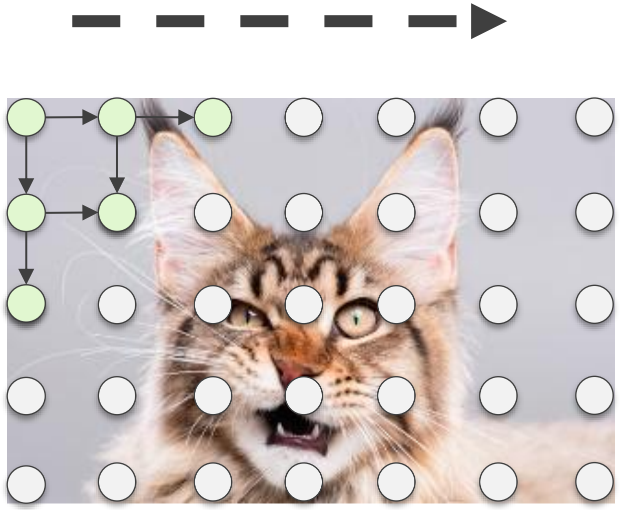

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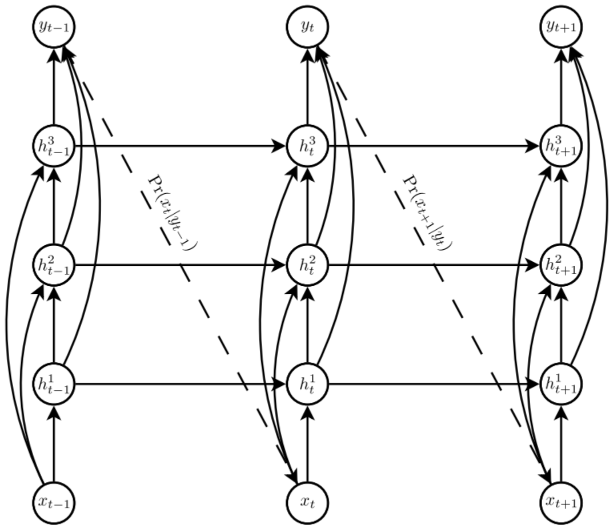

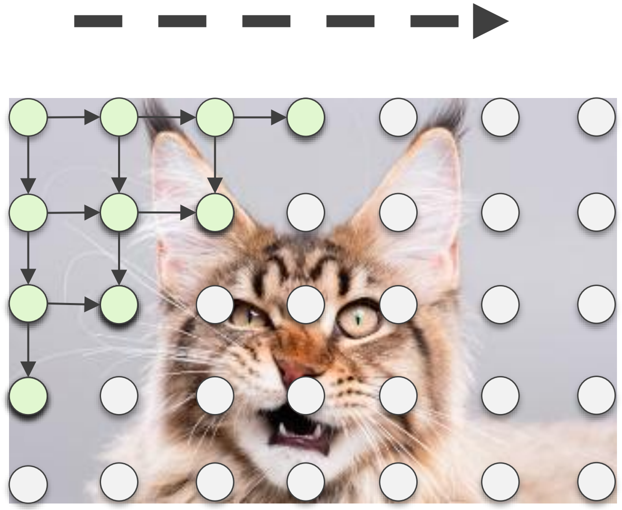

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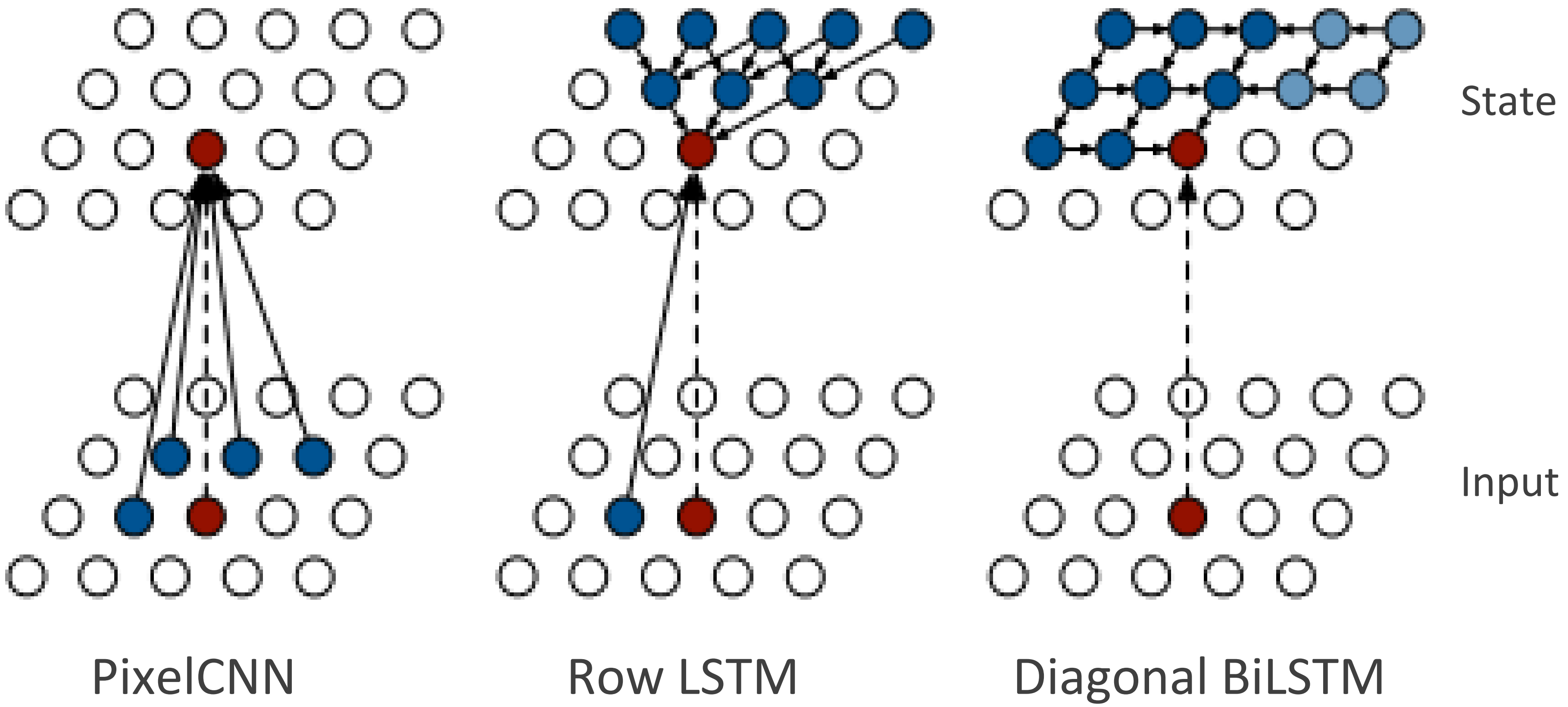
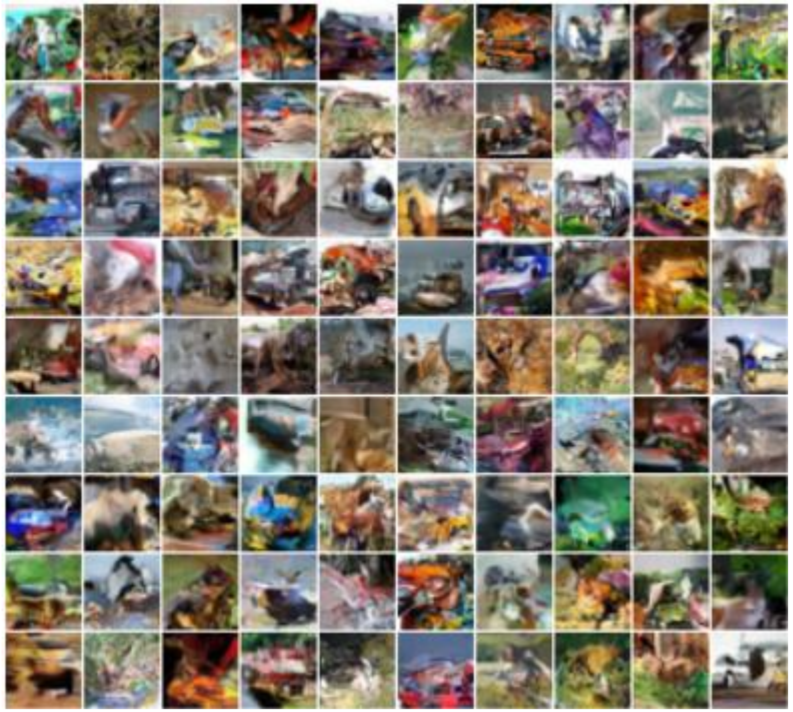
Generating Images Pixel by Pixel

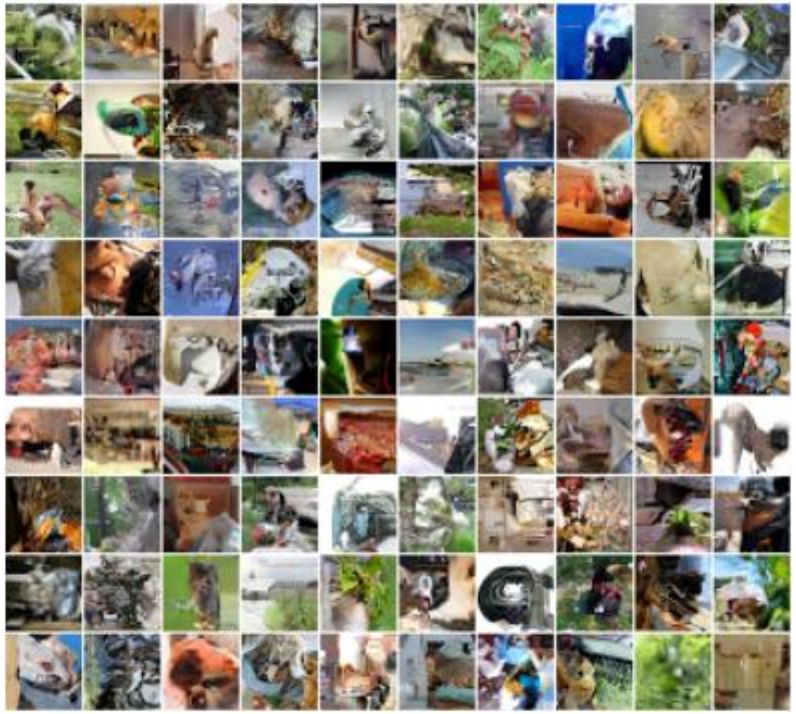


A. van der Oord et al., Pixel Recurrent Neural Networks, 2016

Generating Images Pixel by Pixel - Results



32x32 CIFAR-10



32x32 ImageNet

A. van der Oord et al., Pixel Recurrent Neural Networks, 2016

Variational Autoencoders

From Visible to Latent Information

With only visible information, we try to learn the θ parameterized model distribution

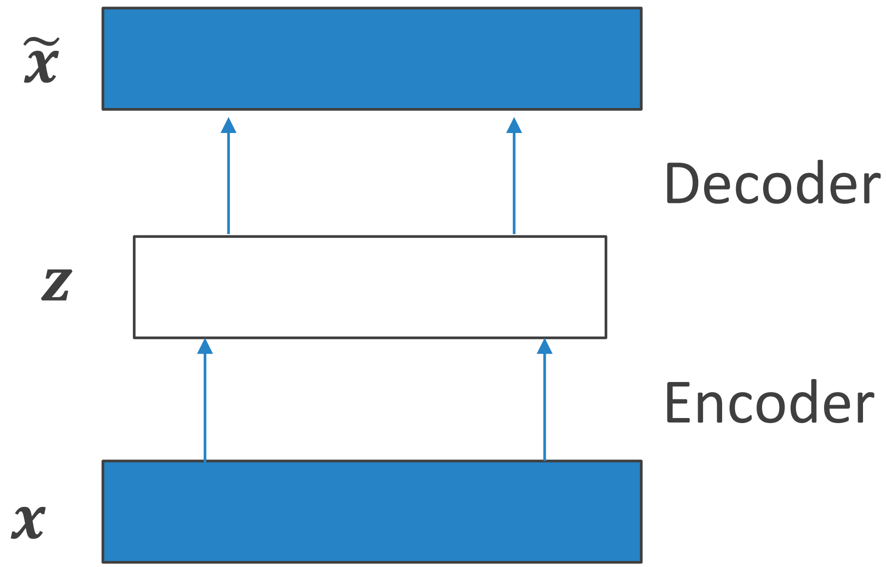
$$P_\theta(\boldsymbol{x}) = \prod_{i}^{N} P_\theta(x_i | x_1, \dots, x_{i-1})$$

Now we introduce a latent process regulated by unobservable variables $\boldsymbol{z}$

$$P_\theta(\boldsymbol{x}) = \int P_\theta(\boldsymbol{x}|\boldsymbol{z}) P_\theta(\boldsymbol{z}) d\boldsymbol{z}$$

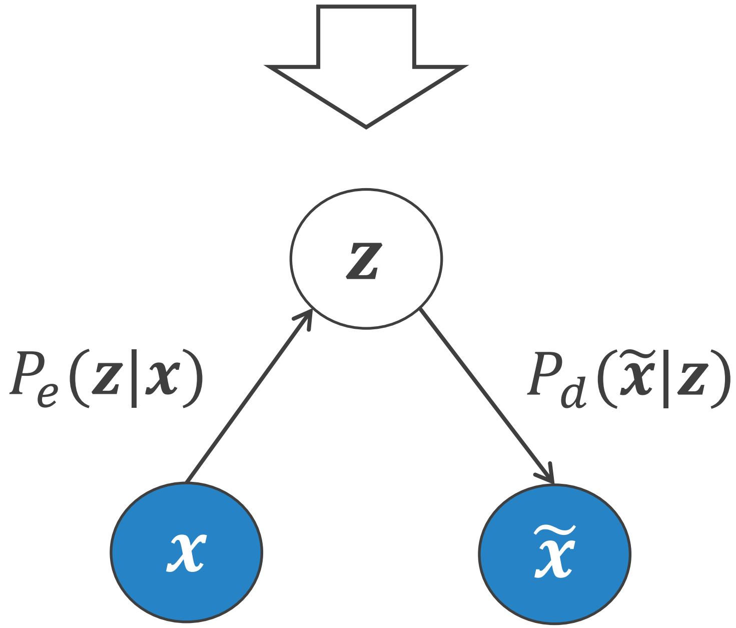
Typically, intractable for nontrivial models (cannot be computed for all $\boldsymbol{z}$ assignments)

A Neural Network with Latent Variables?



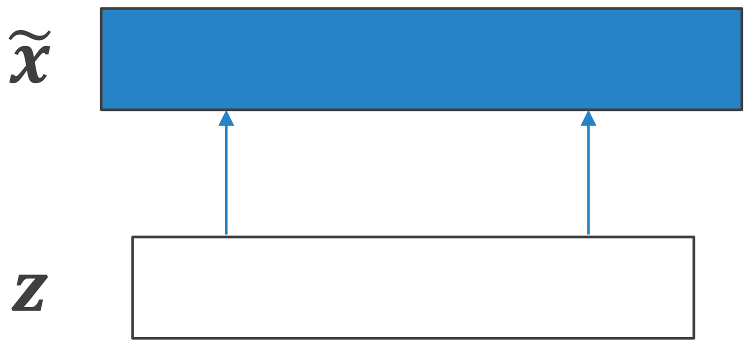
Autoencoder (AE)
neural networks

It is not difficult to cast a probabilistic twist on AE (by making encoder-decoder maps probabilistic)



A Deeper Probabilistic Push

As an additional push in the probabilistic interpretation, we assume to be able to generate the reconstruction from a sampled latent representation

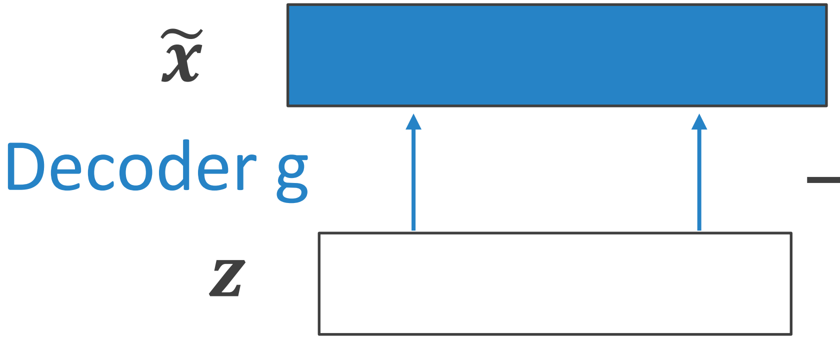


Sample from the true conditional $P(\tilde{\boldsymbol{x}}|\boldsymbol{z})$

Sample latent variables from the true prior $P(\boldsymbol{z})$

Of course we don't have access to the true distributions, so how do we approximate them?

Variational Autoencoders (VAE) – The Catch



Represent the $P(\tilde{\boldsymbol{x}}|\boldsymbol{z})$ distribution through a neural network g (remember the denoising autoencoder)

At training time sample **z** conditioned on data **x** and train the decoder g to reconstruct **x** itself from **z**

Sample $\boldsymbol{z}$ from a simple distribution such as a Gaussian

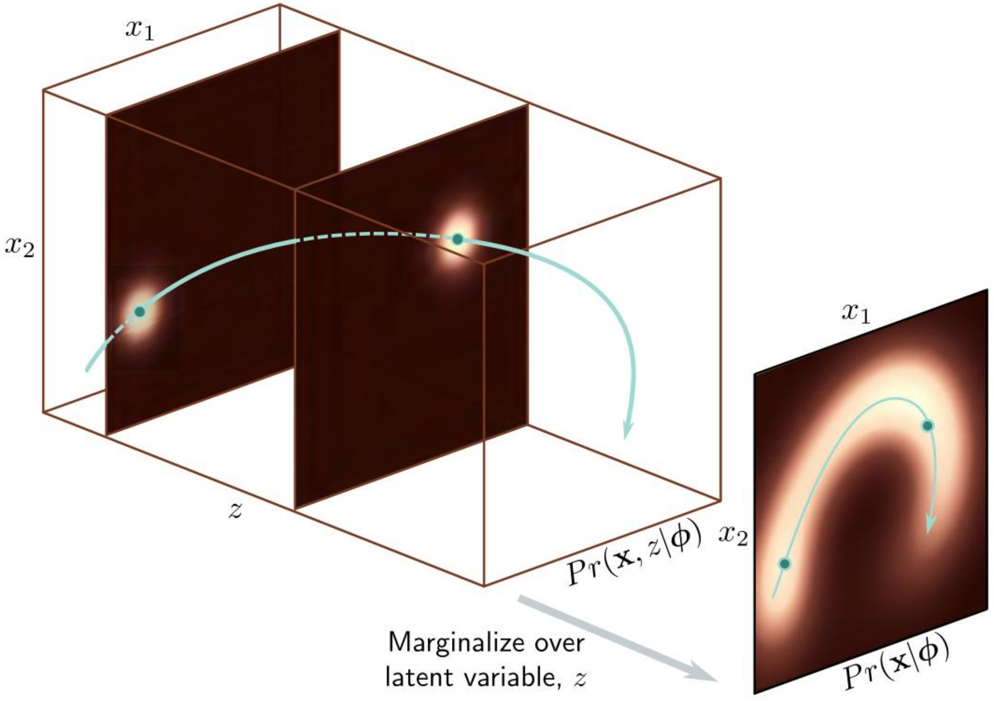
$$z \sim \mathcal{N}(\mu(\boldsymbol{x}), \sigma(\boldsymbol{x}))$$

VAE Training

Ideally, one would like to train maximizing

$$L(D) = \prod_{i=1}^{N} P(\boldsymbol{x_i})$$

$$= \prod_{i=1}^{N} \int P(\boldsymbol{x_i}|\boldsymbol{z})P(\boldsymbol{z})d\boldsymbol{z}$$



VAE Training – Is it all this easy?

Ideally, one would like to train maximizing

$$L(D) = \prod_{i=1}^{N} P(\boldsymbol{x}_i)$$

$$= \prod_{i=1}^{N} \int P(\boldsymbol{x}_i|\boldsymbol{z})P(\boldsymbol{z})d\boldsymbol{z}$$

Unfortunately for you: no!

Intractable

⇩

Variational approximation

Variational Approximation

The revenge of the ELBO (Evidence Lower BOund)

$$\log P(x|\theta) \geq \mathbb{E}_Q[\log P(x,z)] - \mathbb{E}_Q[\log Q(z)] = \mathcal{L}(x,\theta,\phi)$$

Maximizing the ELBO allows approximating from below the intractable log-likelihood $\log P(x)$

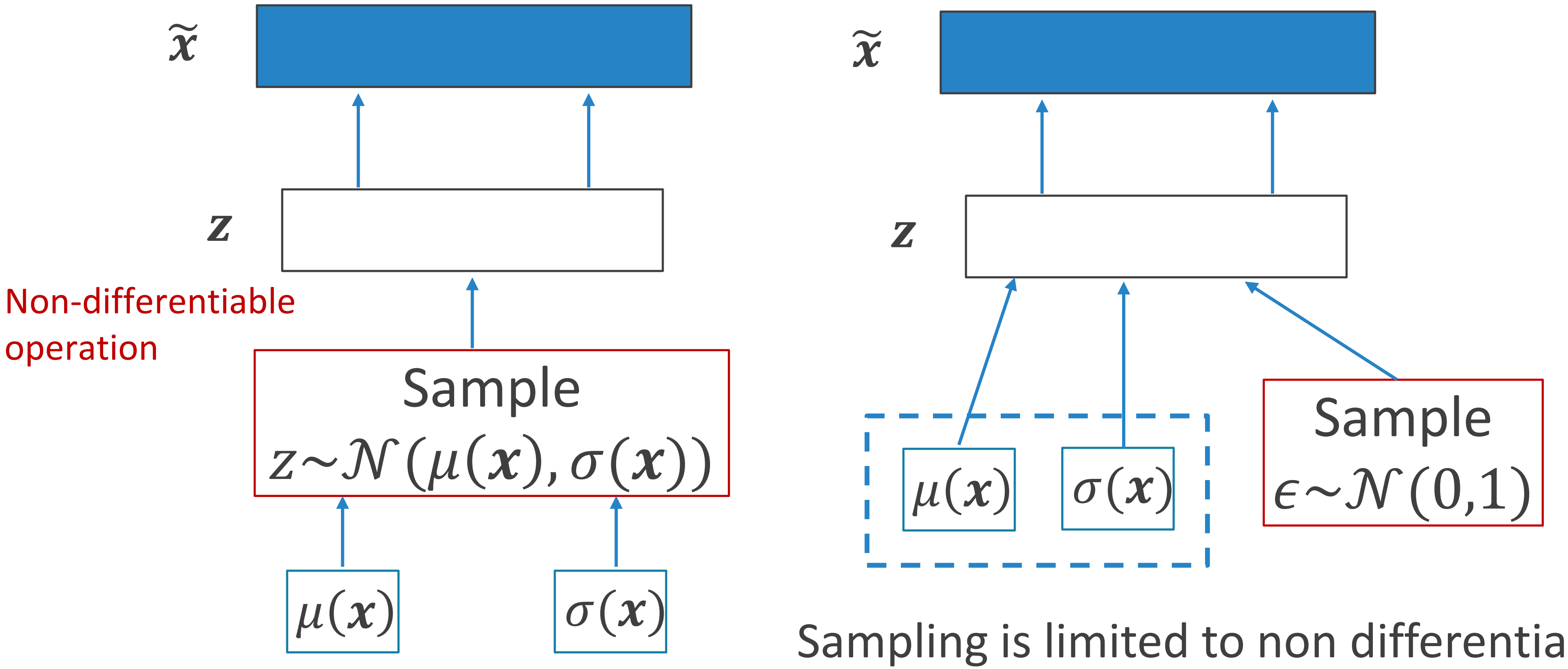
$$\mathcal{L}(x,\theta,\phi) = \mathbb{E}_Q[\log P(x|z)] + \underbrace{\mathbb{E}_Q[\log P(z)] - \mathbb{E}_Q[\log Q(z)]}_{-KL(Q(z|\phi)||P(z|\theta))}$$

Decoder estimate of the reconstruction (based on a sampled z)

(It is not differentiable!)

Need a Q(z) function to approximate P(z)

Reparameterization Trick

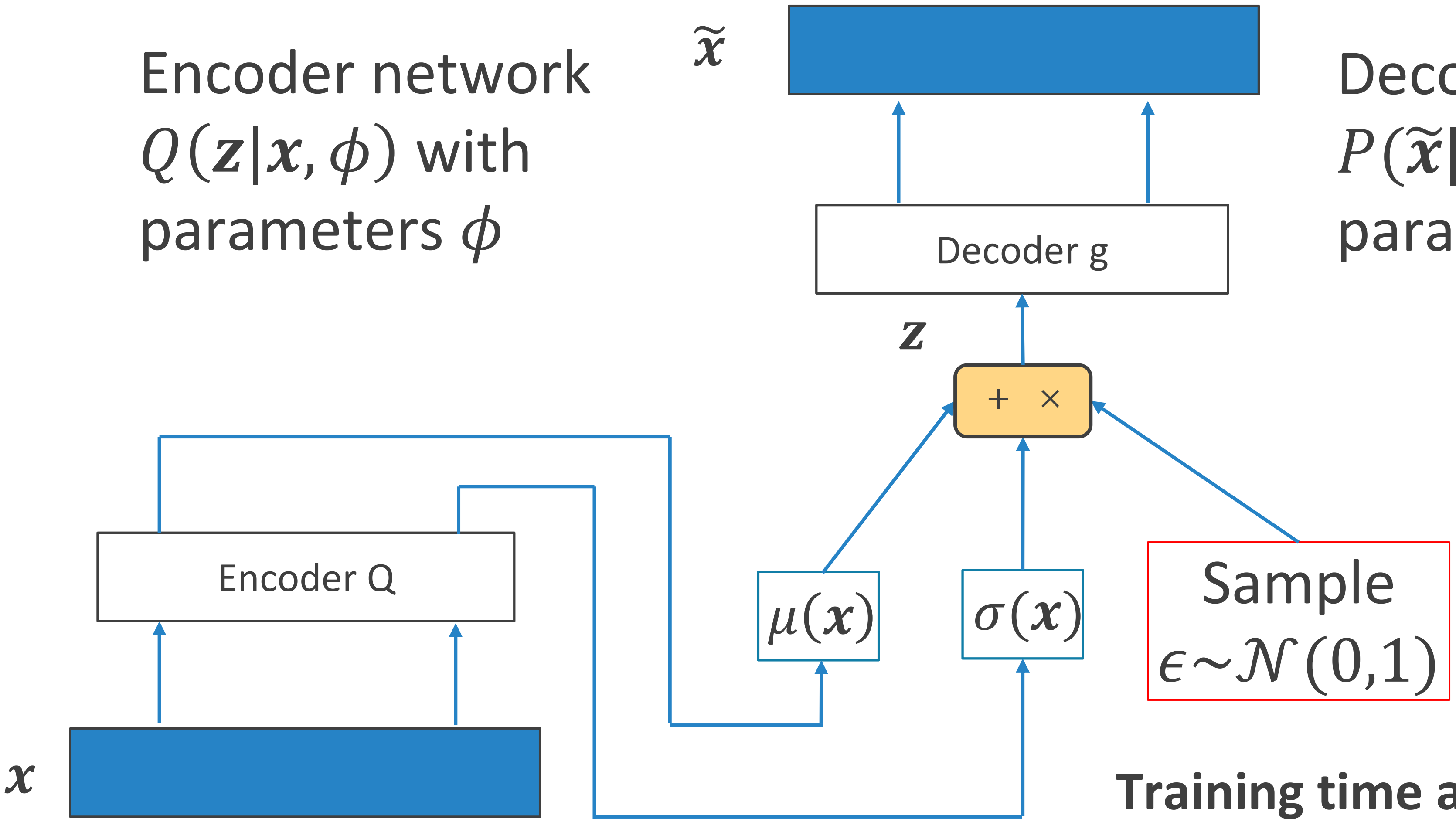


Sampling is limited to non differentiated variable $\epsilon \Longrightarrow$ Can backpropagate

Variational Autoencoder – The Full Picture

Encoder network $Q(\boldsymbol{z}|\boldsymbol{x}, \phi)$ with parameters ϕ

Decoder network $P(\tilde{\boldsymbol{x}}|\boldsymbol{z}, \theta)$ with parameters θ



Training time architecture

VAE Training

Training is performed by backpropagation on θ, ϕ to optimize the ELBO

reconstruction

$$\mathcal{L}(x,\theta,\phi) = \mathbb{E}_Q\left[\log P(x|z = \mu(x) + \sigma^{1/2}(x) * \epsilon, \theta)\right]$$
$$-KL(Q(z|x,\phi)||P(z|\theta))$$

regularization

Can be computed in closed form when both Q(z) and P(z) are Gaussians

$$KL(\mathcal{N}(\mu(x),\sigma(x))\ ||\mathcal{N}(0,1)\ )$$

Train the encoder to behave like a Gaussian prior with zero-mean and unit-variance

VAE Loss – Another view on differentiability

In principle we would like to optimize the following loss by SGD

$$\mathbb{E}_{X\sim D}[\mathbb{E}_{z\sim Q}[\log P(x|z)] - KL(Q(z|x,\phi)||P(z))]$$

which can be rearranged following the reparametrization trick

$$\mathbb{E}_{X\sim D}[\mathbb{E}_{\epsilon\sim\mathcal{N}(0,1)}\left[\log P(x|z=\mu(x)+\sigma^{1/2}(x)*\epsilon,\theta)\right] - KL(Q(z|x,\phi)||P(z))]$$

No expectation is w.r.t distributions that depend on model parameters

⇒ We can move gradients into them

Information Theoretic Interpretation

$$\mathbb{E}_{X\sim D}[\mathbb{E}_{z\sim Q}[\log P(x|z)] - KL(Q(z|x,\phi)||P(z))]$$

Number of bits required to reconstruct x from z under the ideal encoding (i.e. $Q(z|x)$ is generally suboptimal)

Number of bits required to convert an uninformative sample from $P(z)$ into a sample from $Q(z|x)$

Information gain - Amount of extra information that we get about X when z comes from $Q(z|x)$ instead of from $P(z)$

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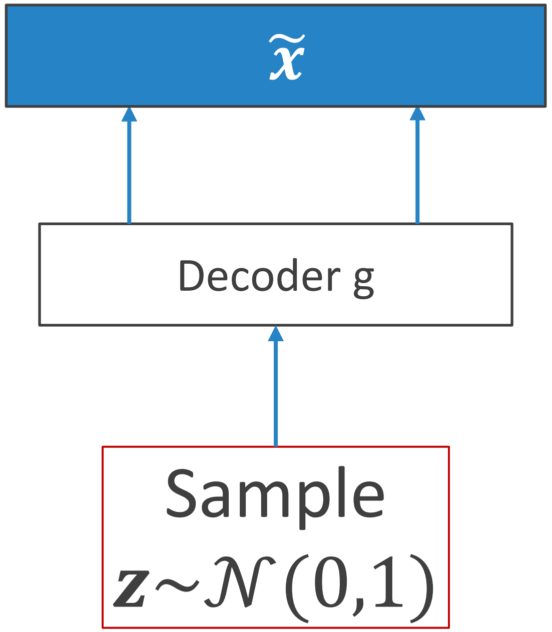
Sampling the VAE (a.k.a. testing)

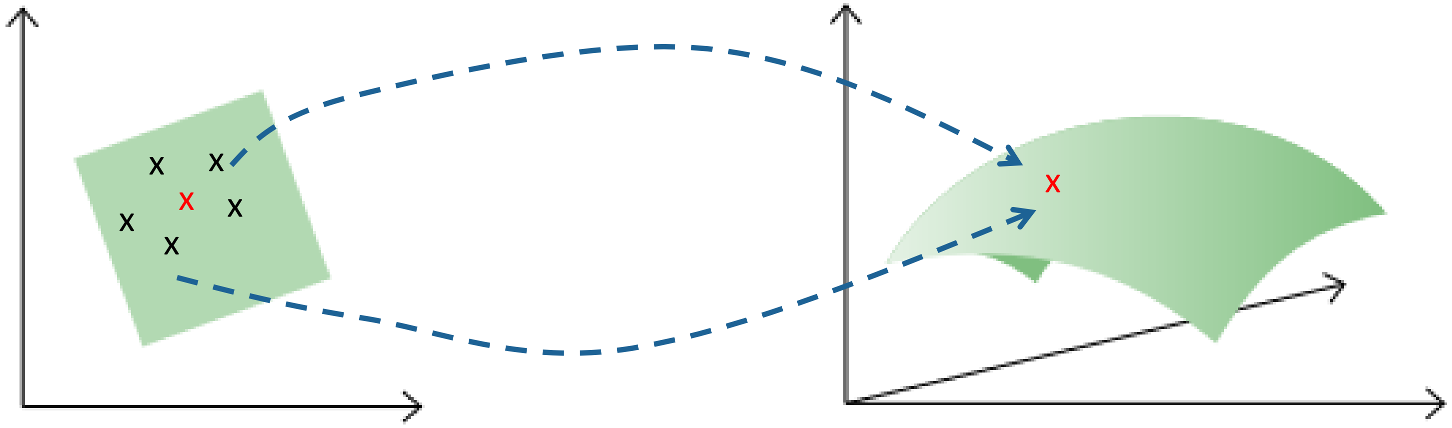


At test time detach the encoder, sample a random encoding and generate the sample as the corresponding reconstruction

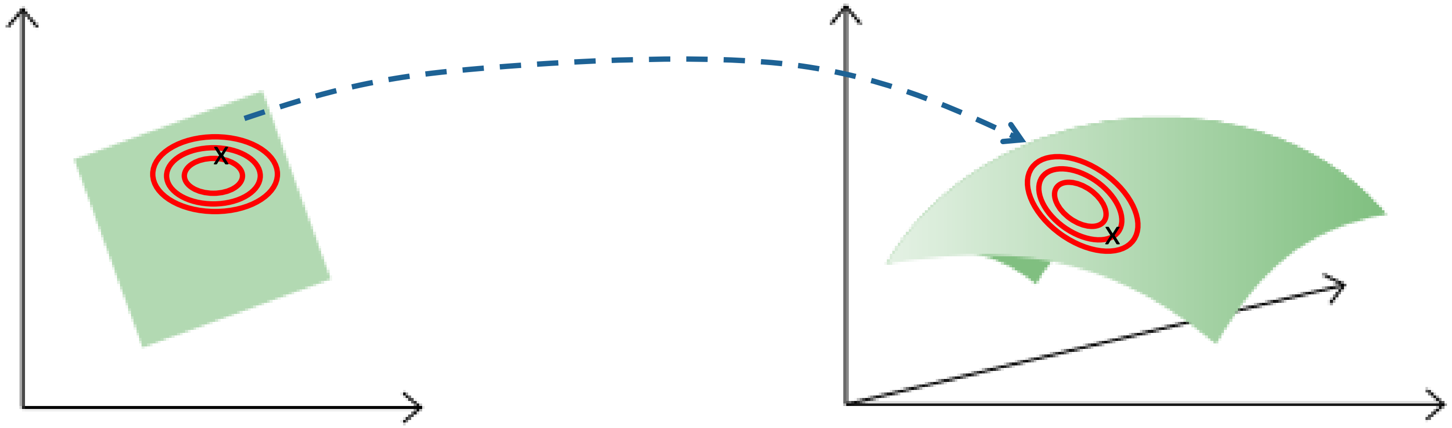
VAE vs Denoising/Contractive AE

Contractive AE

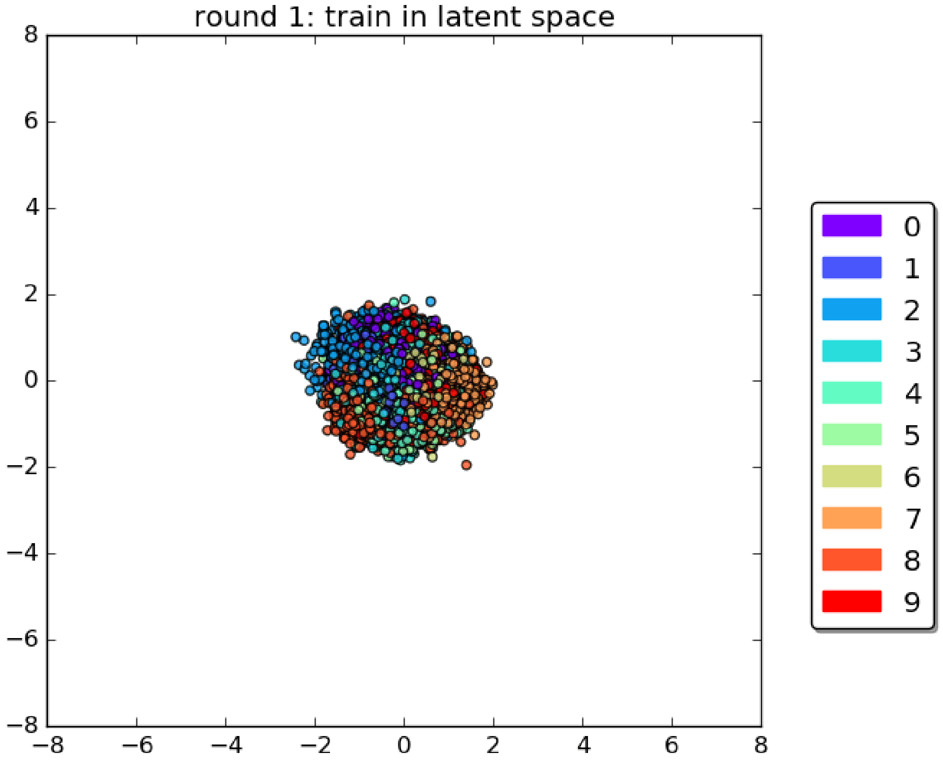
x noisy samples x original sample



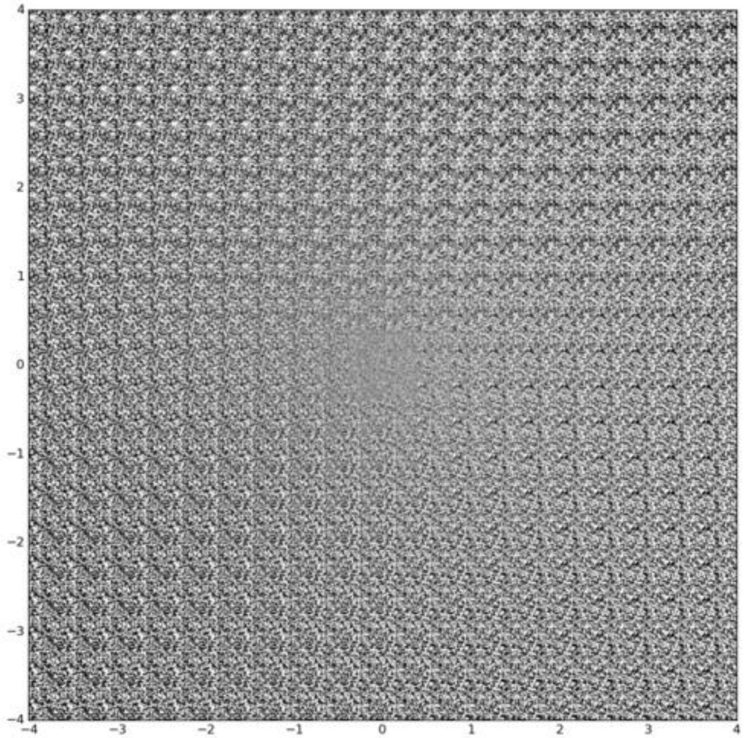
Variational AE



VAE Examples - Digits



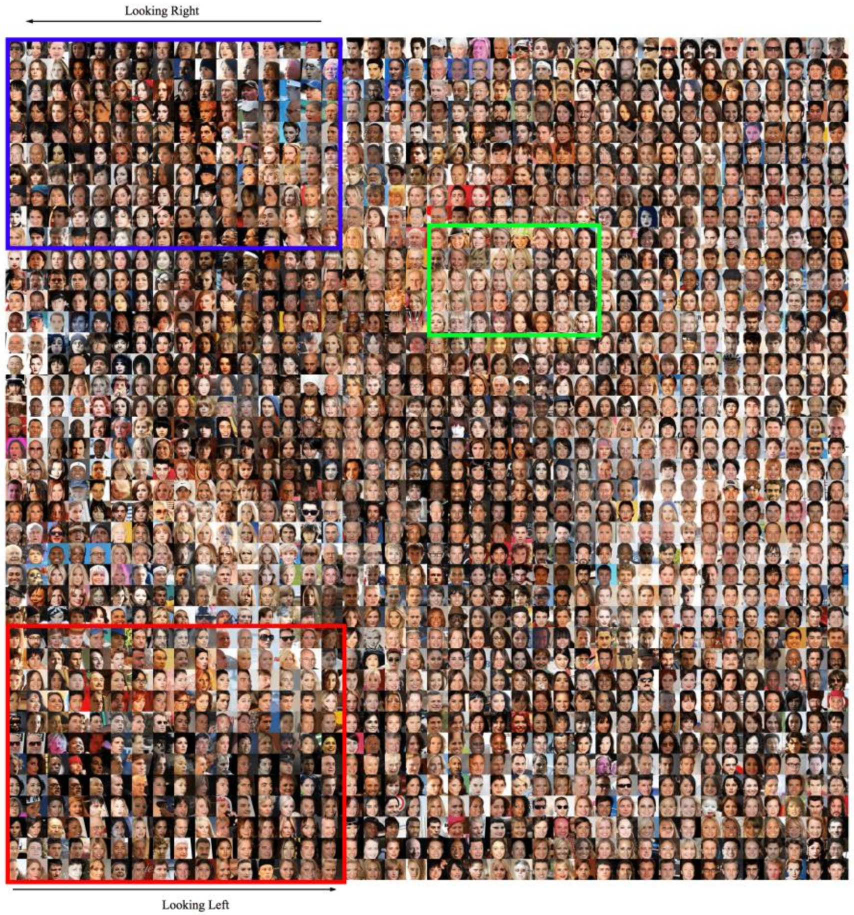
Organization of data in the latent space

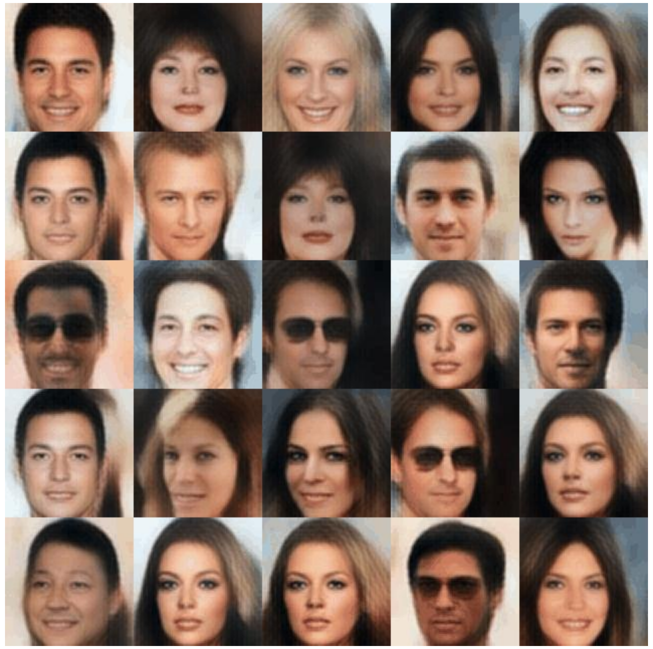


Reconstruction of points sampled from latent space

Image credits @ fastfowardlabs.com

VAE Examples - Faces

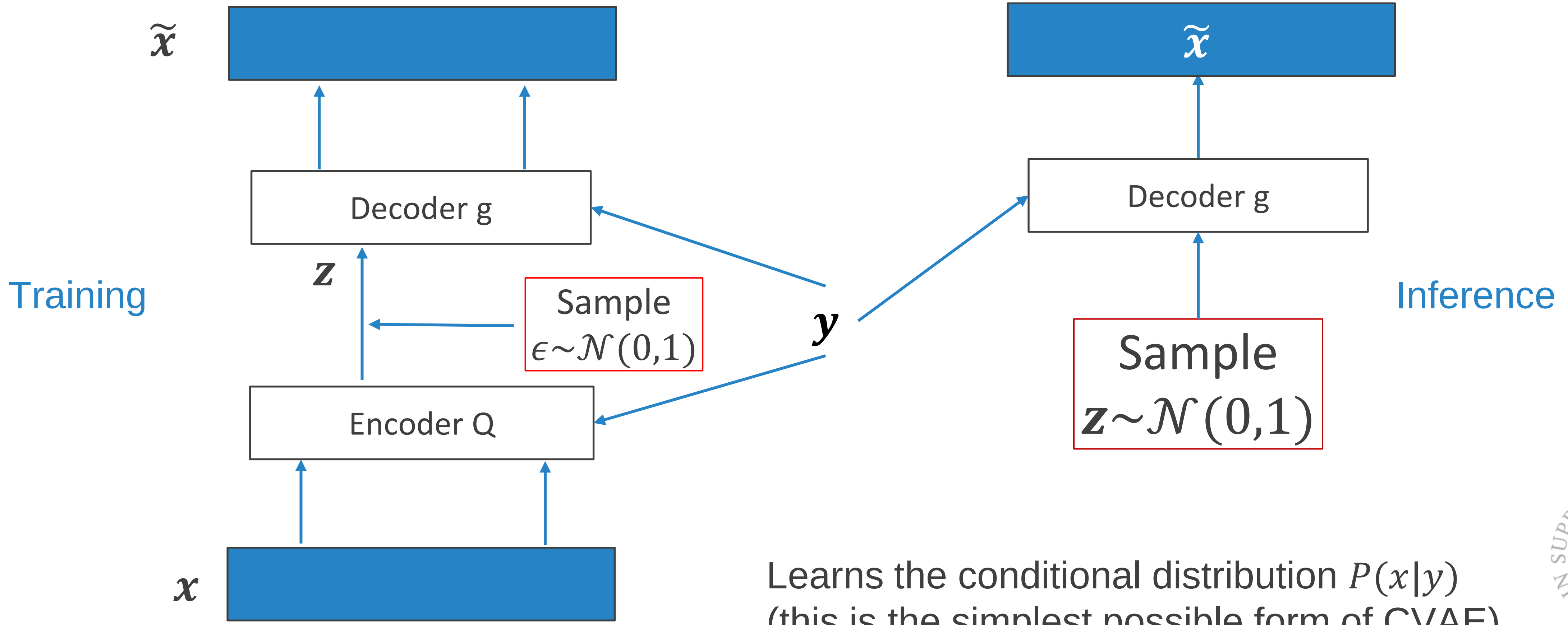




Latent space interpolation

Hou et al, Deep Feature Consistent Variational Autoencoder, 2017

Conditional Generation (CVAE)



Learns the conditional distribution $P(x|y)$
(this is the simplest possible form of CVAE)

Take Home Messages

- PixelRNN/ PixelCNN – Learn explicit distributions by optimizing exact likelihood
  - Yields good samples and excellent likelihood estimates
  - Inefficient sequential generation
- VAE – Learn complex distributions over latent variables through a variational approximation using neural networks
  - Learns a latent representation useful for inference
  - Can lead to poor generated sample quality

Next Lecture

- Learning a sampling process
- Generative adversarial networks
- Hybrid Variational-Adversarial approaches