Ph.D. course: ***A basic and concise introduction to Topological Data Analysis***

COURSE PRESENTATION

- This course will introduce some basic concepts of Computational Topology and Topological Data Analysis. The focus will primarily be on discrete and combinatorial aspects, minimizing topological elements that may be less accessible to computer science students. The details of the proofs will usually be omitted. The aim of the course is to present, in a relatively simple and concise manner, the basic elements of a theory that allows the mathematical formalization of the concept of "shape of data."

- The lectures will be held simultaneously in person and online, and they will be recorded. The recordings of the lessons will be available on the course's Teams page.

- Lecture notes will be made available on Teams and updated progressively throughout the course. These notes contain significantly more material than what will be covered in the course and should not be considered as a description of the exam syllabus.

- Another useful reference (freely available online in its preprint version) is:

  Herbert Edelsbrunner and John Harer. Computational Topology - an Introduction. American Mathematical Society, 2010.

- Each Ph.D. student requiring formal acknowledgment of the course will need to schedule a date with me for an oral exam. During this exam, students will be required to present either a topic covered in the course (e.g., a theoretical result) or a scientific paper related to TDA, chosen at their discretion and communicated to me via email prior to the exam.

- Office hours are available by appointment via email.

Patrizio Frosini

COURSE SYLLABUS AND SCHEDULE

Lecturer: Patrizio Frosini (UNIPI, patrizio.frosini@unipi.it)
Lectures in Sala Seminari Est and online, from 11am to 1pm.

JAN 8th: Equivalence and non-equivalence of data under the action of a group of transformations. Simplicial complexes as a generalization of graphs and as geometric representations of data described by point clouds in Euclidean spaces.

JAN 10th: Simplicial homology groups as a method for representing the "shape" of a simplicial complex derived from a point cloud.

JAN 13th: Computation of homology groups.

JAN 15th: Adapting homology to the observer's point of view and accounting for noise: an introduction to persistent homology and persistence diagrams. Stability of persistence diagrams in the presence of noise.

JAN 17th: Applications of persistent homology.

JAN 20th: From the shape of data to the shape of observers: the concept of a Group Equivariant Non-Expansive Operator (GENEO) and the problem of approximating observers in the space of GENEOs.

JAN 22th: GENEOs as a geometric method for reducing the number of parameters in neural networks and enhancing their interpretability.

JAN 24th: Application of GENEO theory to identify pockets in proteins and the implementation of GENEO networks for geometric machine learning.