Definite matrices

$A = A^* \in \mathbb{C}^{n \times n}$, quadratic form $f(x) := x^* A x$. A is called...

Name	Definition
Positive definite	$x^* A x > 0$ for all $x \neq 0$
Positive semidefinite	$x^* A x \geq 0$ for all x
Indefinite	$x^* A x$ can be > 0 or < 0
Negative semidefinite	$x^* A x \leq 0$ for all x
Negative definite	$x^* A x < 0$ for all $x \neq 0$

Definiteness and eigenvalues

Conditions on eigenvalues for definiteness:

Name	Definition	Eigenvalues
Positive definite	$x^*Ax > 0$ for all $x \neq 0$	$\lambda > 0$ for all eigvls
Positive semidefinite	$x^*Ax \geq 0$ for all x	$\lambda \geq 0$ for all eigvls
Indefinite	x^*Ax can be > 0 or < 0	Some > 0, some < 0
Negative semidefinite	$x^*Ax \leq 0$ for all x	$\lambda \leq 0$ for all eigvls
Negative definite	$x^*Ax < 0$ for all $x \neq 0$	$\lambda < 0$ for all eigvls