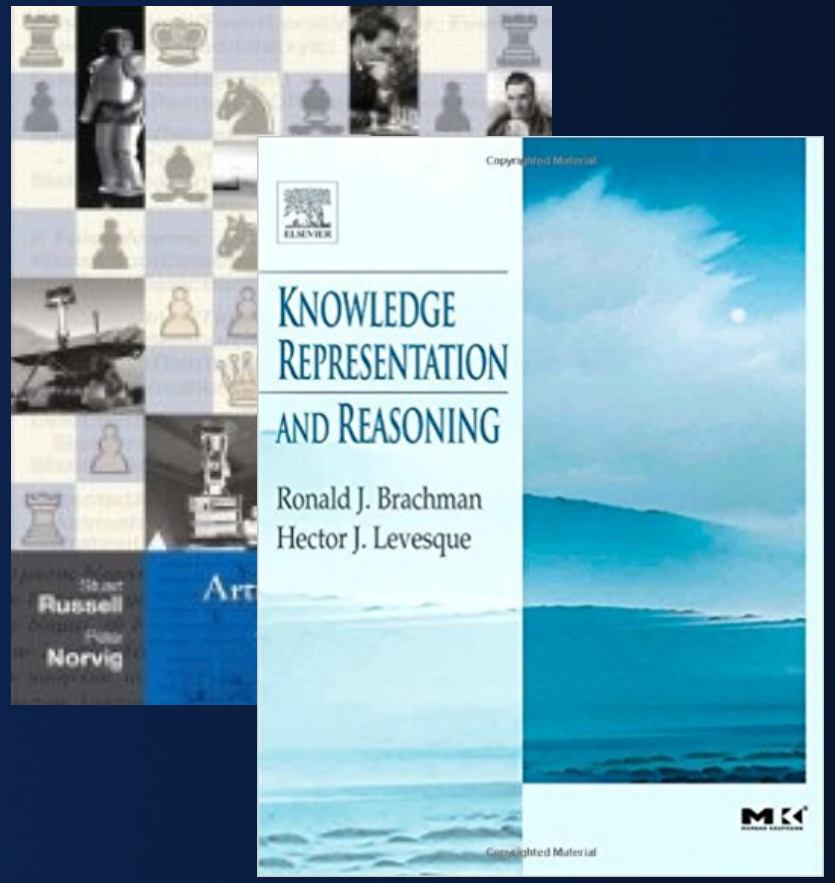


AI Fundamentals: rule-based systems

Maria Simi

Rule-based systems: the basics

LESSON 1- SLD RESOLUTION

Rules are everywhere

1. Expert systems
2. Natural language processing, e.g. chatbots
3. Rule-based programming
4. Business logics in organizations
5. End user programming, e.g. in domotics

Resolution theorem proving

Given the the fundamental problem KB $\models \alpha$ an equivalent problem is "KB $\cup \neg\alpha$ is unsatisfiable"

This can be solved by a deductive system showing that $\mathrm{KB} \cup \neg\alpha \vdash \{\}$, where $\{\}$ is the empty clause meaning *False*.

Resolution by refutation is a method which is *correct* and *complete*:

1. transform the KB **in clausal form** (a conjunction of disjunction of literals)
2. add to KB the negation of the goal in clausal form
3. use the **resolution rule** as unique inference rule.

This strategy works for PROP and FOL with different complexity results:

1. It is decidable and NP-complete for PROP
2. It is semi-decidable for FOL.

Resolution rule

Clauses are set of literals, i.e. atomic formulas or their negation: $\{p_1, p_2, \ldots p_k\}$

Resolution rule for PROP (c_1 and c_2 are clauses):

$$\frac{c_1 \cup \{p\} \qquad c_2 \cup \{\neg p\}}{c_1 \cup c_2} \qquad (\textit{the resolvent})$$

In a resolution refutation we aim to deduce the empty clause:

$$\frac{\{p\} \qquad \{\neg p\}}{\{\,\}}$$

Resolution rule for FOL:

$$\frac{c_1 \cup \{p\} \qquad c_2 \cup \{\neg q\}}{[\,c_1 \cup c_2]\,\gamma}$$

and MGU$(p, q) = \gamma$ and γ is not *fail*

a fundamental operation is **unification**

Resolution in the predicate calculus

Unification: is a process to determine whether two expressions can be made identical by a substitution of terms to variables.

The result is the unifying substitution, the **unifier**, or FAIL, if the expressions are not unifiable.

For example:

$\{P(f(y), A), Q(B, C)\}$ $\{\neg P(x, A), R(x, C), S(A, B)\}$ x and y are variables

With the substitution $\{x/f(y)\}$ the literal $\neg P(x, A)$ becomes $\neg P(f(y), A)$, and it is possible to apply the resolution rule.

The resolvent is: $\{Q(B, C), R(f(y), C), S(A, B)\}$

Given two expressions there may be different substitutions that make them identical. We are interested in computing the **most general unifier (MGU)**, the one that does only the essential instantiations.

Unification algorithm [Martelli, Montanari, 1982]

- Computes the MGU by means of a **rule-based** equation-rewriting system
- Initially the working memory (WM) contains the equality of the two expressions to be unified
- The rules modify the equations in the WM
- The algorithm terminates with failure or when there are no applicable rules (**success**)
- At the end, if there is no failure, the WM contains the MGU.

Note: different from the AIMA unification algorithm but easier to understand.

The rules

1. $f(s_1, \ldots, s_n) = f(t_1, \ldots, t_n) \rightarrow s_1 = t_1, \ldots, s_n = t_n$
2. $f(s_1, \ldots, s_n) = g(t_1, \ldots, t_m) \rightarrow$ *fail* when $f \neq g$ or $n \neq m$
3. $x = x \rightarrow$ remove equation
4. $t = x \rightarrow x = t$ bring variable to the left
5. $x = t$, x does not occur in $t \rightarrow$ apply $\{x/t\}$ to other equations
6. $x = t$, t is not x, x occur in $t \rightarrow$ *fail* (*occur check*)

Nota: when comparing two different constants, rule 2 applies, as a special case where $n = m = 0$, and we fail.

Unification algorithm: example 1

Computing the MGU of $P(A, y, z)$ and $P(x, B, z)$

Step 3

$x = A$

$y = B$

MGU!

Unification algorithm: example 2

Computing the MGU of $P(f(x), x)$ and $P(z, z)$

Step 3

$z = f(x)$

$x = f(x)$ rule 6

FAIL!

(occurr check)

Reasoning with Horn clauses

By **limiting expressivity** to only a certain interesting subset of first-order logic, resolution procedures becomes much more manageable.

We limit the degree of uncertainty we can express by considering clauses that have **at most one positive literal** (Horn clauses). Three cases:

1. **Rules**:

   $\{\neg Child, \neg Male , \boldsymbol{Boy}\}$ is logically equivalent to

   $Child \wedge Male \Rightarrow Boy$ which has a natural interpretation as a rule.

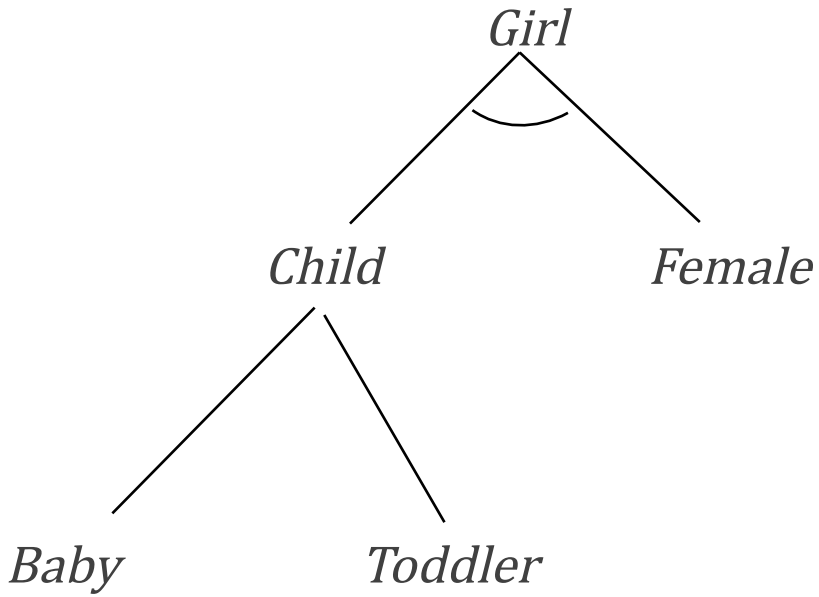
2. **Facts**: $\{\boldsymbol{Child}\}$
3. **Goals or queries**: $\{\neg Boy\}$ only negative literals (**negative clauses**)

Propositional Horn clauses have **linear-time** deduction algorithms.

Goal trees are and-or trees

1. *Toddler*
2. *Baby* ⇒ *Child*
3. *Toddler* ⇒ *Child*
4. *Child, Male* ⇒ *Boy*
5. *Infant* ⇒ *Child*
6. *Child, Female* ⇒ *Girl*
7. *Female*

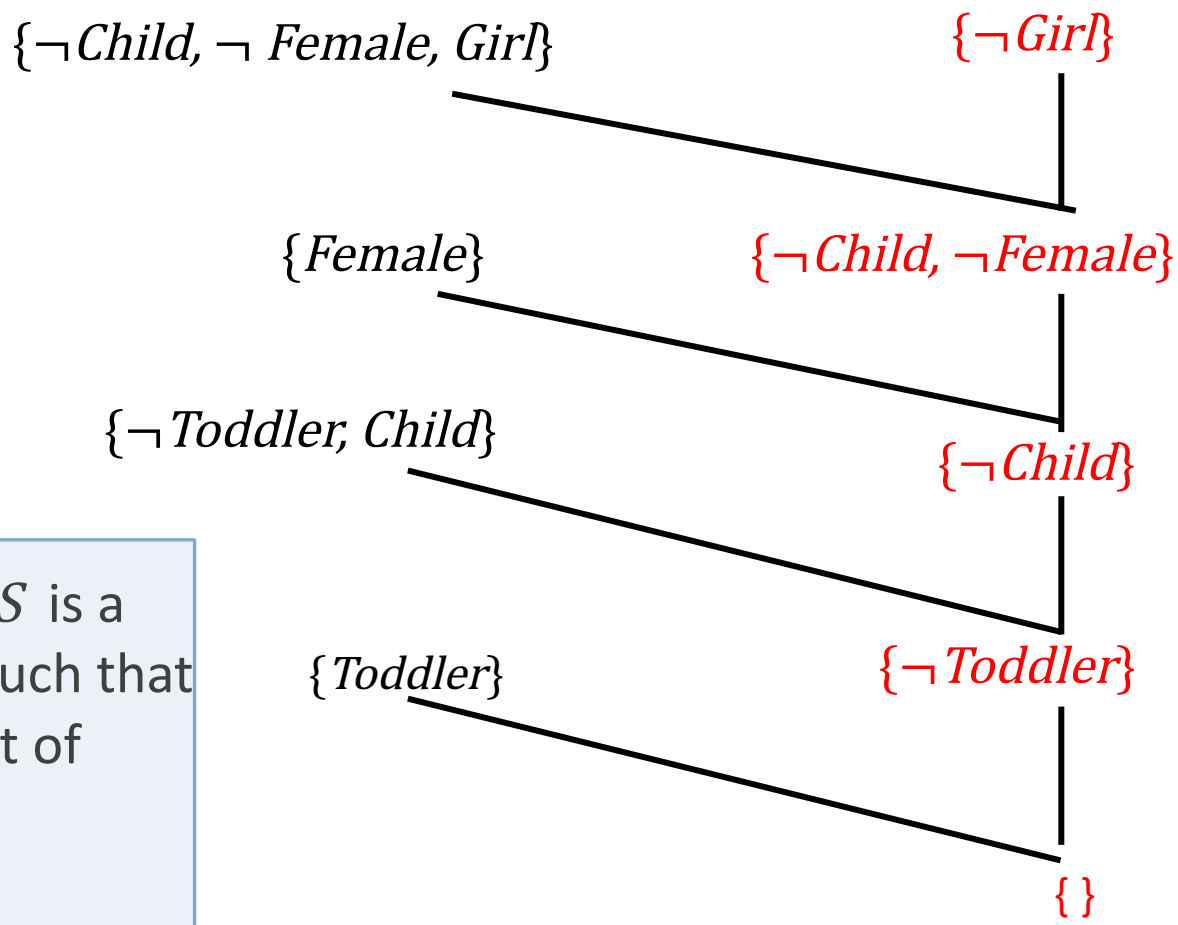
GOAL: *Girl*



Example of SLD resolution

$\{Toddler\}$
$\{\neg Baby, Child\}$
$\{\neg Toddler, Child\}$
$\{\neg Child, \neg Male, Boy\}$
$\{\neg Infant, Child\}$
$\{\neg Child, \neg Female, Girl\}$
$\{Female\}$
GOAL: $\{\neg Girl\}$

A SLD derivation of a clause c from S is a sequence of clauses $c_1, c_2, \ldots, c_n$, such that $c_n = c$, $c_1 \in S$, and c_{i+1} is a resolvent of c_i and some clause in S.
$S \vdash_{SLD} A$ iff $S \vdash A$



Logic programs

A logic program is a set of **definite** Horn clauses (facts and rules).

A.

A :– $\boldsymbol{B}_1$, B_2, … , B_n. (A *head*, $\boldsymbol{B}_1$, B_2, … , B_n *body*)

***Declarative* interpretation**

A is true.

A is true **if** $\boldsymbol{B}_1$, B_2, … , B_n are all true.

The goal (query) is a negative clause whose logical meaning is $\neg(G_1 \wedge G_2 \wedge \ldots \wedge G_\mathrm{k})$

written as ?– G_1, G_2, … , G_k

Procedural interpretation

The head of a rule can be seen as a function call and the body as functions to be called in sequence. When they all return the main procedure returns.

Example of logic program

1. parent(X, Y) :– father(X, Y).
2. parent(X, Y) :– mother(X, Y).
3. ancestor(X, Y) :– parent(X, Y).
4. ancestor(X, Y) :– parent(X, Z), ancestor(Z, Y).
5. father(john, mark).
6. father(john, luc).
7. mother(lia, john).

8. ?– ancestor(lia, mark) *the negation of the goal*

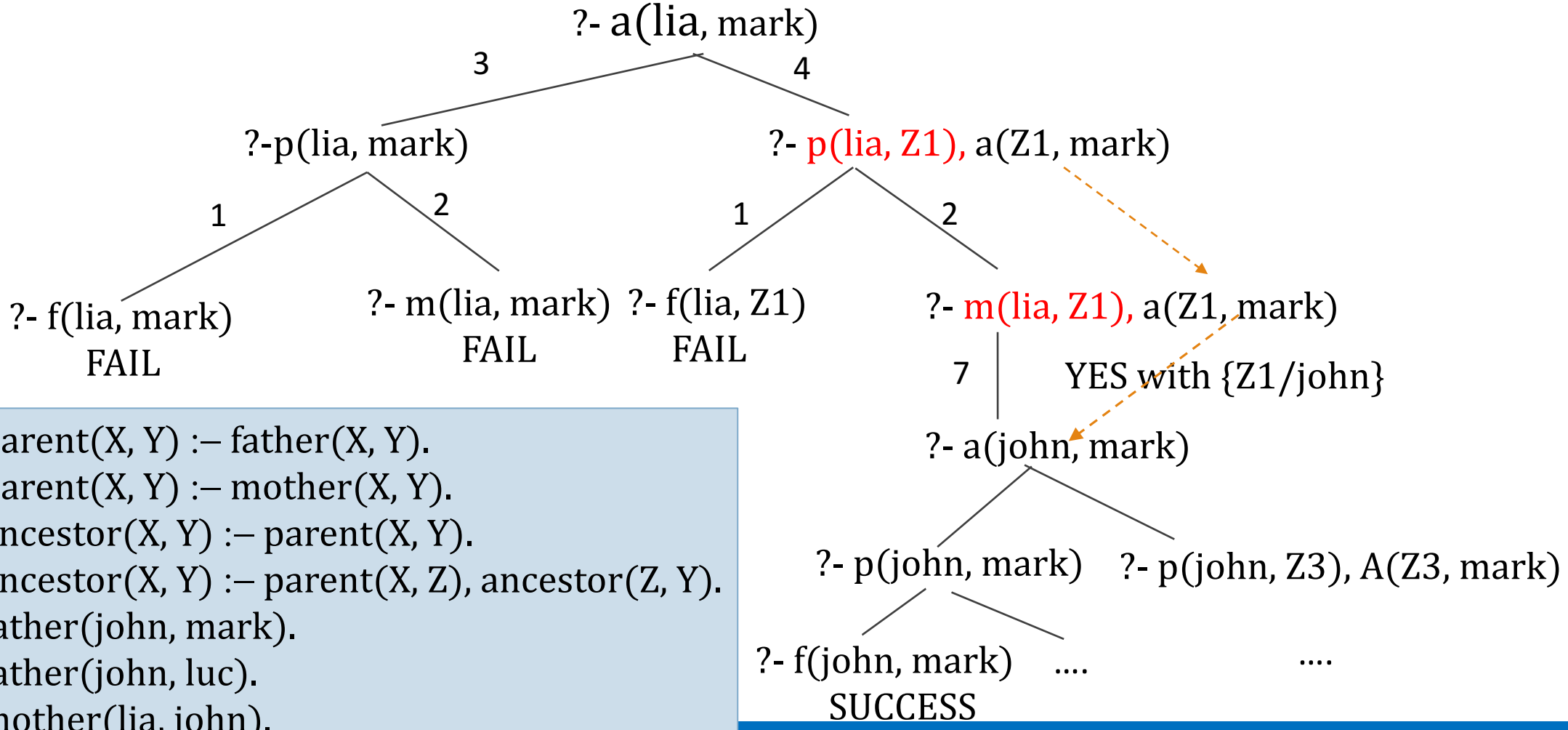
SLD resolution

Given a logic program and a goal $G_1, G_2, \ldots, G_k$ the SLD goal tree is constructed as follows. Each node of the tree corresponds to a conjunctive goal to be solved.

- The root node is ?- $G_1, G_2, \ldots, G_k$
- Let ?– $G_1, G_2, \ldots, G_k$ a node in the tree; the node successors are obtained by considering the facts and rules in the program whose head is unifiable with G_1
  - If A :– $B_1, \ldots, B_m$ is a rule and γ = MGU(A, G_1), a descendent is the new goal ?– $(B_1, \ldots, B_m, G_2, \ldots, G_k)\gamma$
  - If A is a fact and γ = MGU(A, G_1), a descendent is the new goal ?– $(G_2, \ldots, G_k)\gamma$

  Note: variables in rules are renamed before using them.
- Nodes that correspond to empty clauses are s**uccesses**.
- Nodes without successors are **failures**.

SLD tree for goal *ancestor(lia, mark)*



1. parent(X, Y) :– father(X, Y).
2. parent(X, Y) :– mother(X, Y).
3. ancestor(X, Y) :– parent(X, Y).
4. ancestor(X, Y) :– parent(X, Z), ancestor(Z, Y).
5. father(john, mark).
6. father(john, luc).
7. mother(lia, john).

Computed answers

In addition to ground goals such as ?-$ancestor(lia, mark)$ whose answer is YES or NO, we can also have goals with variables such as

?- $ancestor(X, mark)$.

The logical meaning of the query is:

$\mathrm{KB} \models \exists X\, ancestor(X, mark)$? *Are there ancestors of Mark?*

$\mathrm{KB} \cup \neg\exists X\, ancestor(X, mark)$ unsatisfiable?

$\mathrm{KB} \cup \{\neg ancestor(X, mark)\} \vdash_{\mathrm{RES}} \{\}$?

In this case the expected answer are the values that X is bound to during the resolution proof.

X=lia; X=john

Similarly: *?-* $ancestor(lia, Y)$

returns all the descendants of Lia (Y=john, Y=mark, Y=luc).

SLD resolution strategy

The SLD resolution strategy is complete for definite Horn clauses.

This means that if $P \cup \{\neg G\}$ is unsatisfiable, then at least one of the leaves of the goal tree produces the empty clause (success).

Moreover trying to satisfy the subgoals in the order they appear it is not restrictive, since in the end all of them must be satisfied.

When there are variables in the goal, the substitution that we obtain is the **computed answer**.

Completeness and efficiency are however influenced by:

- the order of expansion of the nodes (the visit strategy)
- the order in which we consider successor nodes at each level
- the order of literals in the body

Your turn

- ✓ Make sure you understand all these premises.
- ✓ Do something useful for the IIA students: implement the rule-based version of UNIFICATION to be used in connection with the AIMA code in file logic.py.
- ✓ Get familiar with the online version of SWISH Prolog (if you prefer, you can install it on your PC).

  https://swish.swi-prolog.org/

References

- ✓ Stuart J. Russell and Peter Norvig. *Artificial Intelligence: A Modern Approach* (3rd edition). Pearson Education 2010 [Chapter 9]