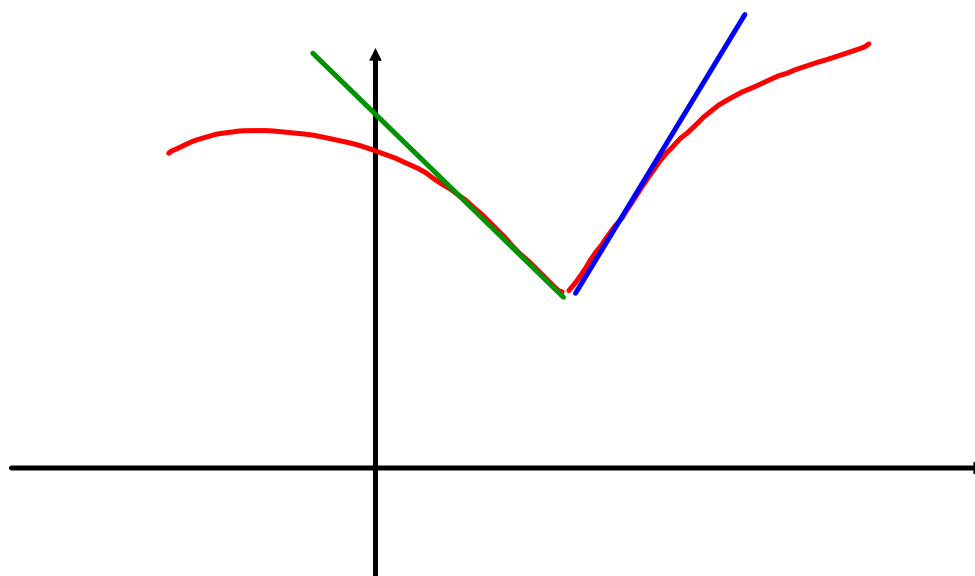


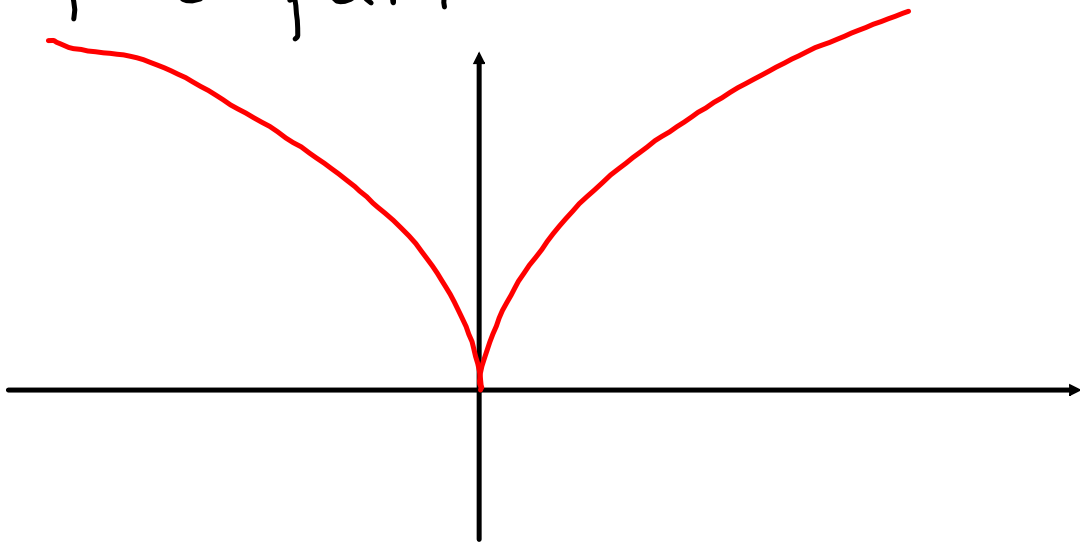
Def: Se
esistono $f'_-(x_0)$ e $f'_+(x_0)$
ma $f'_-(x_0) \neq f'_+(x_0)$
e sono entrambe finite
 $\Rightarrow x_0$ si dice punto angoloso.



Def: Se f è continuo in x_0
e $f'_+(x_0) = +\infty$ e $f'_-(x_0) = -\infty$
o viceversa allora x_0 si
dice punto di cuspidale.

$$E_s : f(x) = \sqrt{|x|}$$

f é par



Teorema .

Se f e g sono derivabili in x_0
allora

1) $f+g$ è derivabile in x_0 e

$$(f+g)'(x_0) = f'(x_0) + g'(x_0)$$

2) $f \cdot g$ è derivabile in x_0 e

$$(fg)'(x_0) = f'(x_0)g(x_0) + f(x_0)g'(x_0)$$

3) Se $f(x_0) \neq 0 \Rightarrow \frac{1}{f}$ è
derivabile in x_0 e

$$\left(\frac{1}{f}\right)'(x_0) = -\frac{f'(x_0)}{[f(x_0)]^2}$$

Oss: Se f è derivabile in x_0
e g è derivabile in x_0 con
 $g(x_0) \neq 0$ allora

$$\left(\frac{f}{g}\right)'(x_0) = \left(f \cdot \frac{1}{g}\right)'(x_0) = f'(x_0) \cdot \frac{1}{g(x_0)} + f(x_0) \left(-\frac{g'(x_0)}{[g(x_0)]^2}\right) =$$

$$= \frac{f'(x_n)g(x_n) - f(x_n)g'(x_n)}{[g(x_n)]^2}$$

Derivata di e^x

$x_0 \in \mathbb{R}$ fissato.

$$\lim_{x \rightarrow x_0} \frac{e^x - e^{x_0}}{x - x_0} =$$

$$\lim_{x \rightarrow x_0} \frac{e^{x_0 + x - x_0} - e^{x_0}}{x - x_0} =$$

$$= \lim_{x \rightarrow x_0} \frac{e^{x_0} \cdot e^{x-x_0} - e^{x_0}}{x - x_0} =$$

$$= \lim_{x \rightarrow x_0} e^{x_0} \frac{e^{x-x_0} - 1}{x - x_0} =$$

$$= e^{x_0} \lim_{t \rightarrow 0}$$

$$= e^{x_0}$$

$$\frac{e^t - 1}{t}$$

$$\begin{aligned} x - x_0 &= t \\ x \rightarrow x_0 &\Rightarrow t \rightarrow 0 \end{aligned}$$

$$\rightarrow 1$$

quindi

$$D(e^x) = e^x \quad \forall x \in \mathbb{R}.$$

$$D(\sin x) = \cos x \quad \forall x \in \mathbb{R}$$

$$D(\cos x) = -\sin x.$$

Oss: Se f è costante
allora $f'(x) = 0 \quad \forall x$.

Oss: Se $k \in \mathbb{R} \Rightarrow$
 $D(kf) = k D(f).$

$$\lim_{x \rightarrow x_0} \frac{kf(x) - kf(x_0)}{x - x_0} =$$

$$= k \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0} = k f'(x_0).$$

$$E_s: \quad D(-\sin x) = -\cos x$$

$$f(x) = \sin x$$

$$f'(x) = \cos x$$

$$f''(x) = D(\cos x) = -\sin x$$

$$f'''(x) = D(-\sin x) = -\cos x$$

$$f^{(4)}(x) = D(-\cos x) =$$

$$= -D(\cos x) = -(-\sin x) = \sin x.$$

$$f^{(5)}(x) = f'(x)$$

in general

$$f^{(k+1)}(x) = f^{(k)}$$

Lo stesso per $\cos x$.

$$\begin{aligned} D(\operatorname{tg} x) &= D\left(\frac{\sin x}{\cos x}\right) = \\ &= \frac{D(\sin x) \cdot \cos x - \sin x D(\cos x)}{(\cos x)^2} = \\ &= \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{(\cos x)^2} = \end{aligned}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} =$$

$$= \frac{1}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = 1 + (\operatorname{tg} x)^2$$

Derivata della funzione
inversa.

$f: (a, b) \rightarrow \mathbb{R}$ derivabile
e strettamente monotona.

Allora f^{-1} è derivabile

$$\text{e } (f^{-1})'(y) = \frac{1}{f'(x)} \quad \text{ovvero } y = f(x).$$

quindi

$$(f^{-1})'(y) = \frac{1}{f'(f^{-1}(y))}$$

$$\text{Es: } f(x) = e^x \quad f'(x) = e^x$$

surto $y = e^x$

$$x = \log y$$

$$\log y = f^{-1}(y)$$

$$D(\log y) = D(f^{-1}(y)) =$$

$$= \frac{1}{f'(f^{-1}(y))} = \frac{1}{e^{f^{-1}(y)}} =$$

$$= \frac{1}{e^{\log y}} = \frac{1}{y}.$$

$$D(\log y) = \frac{1}{y}.$$

Quindi la funzione $\log x$
è derivabile in tutto il
suo dominio $(0, +\infty)$

$$e \quad D(\log x) = \frac{1}{x} .$$

Prop: Se f è derivabile
in x_0 e g è derivabile in
 $f(x_0)$ allora $g \circ f$ è derivabile
in x_0 e

$$(g \circ f)'(x_0) = g'(f(x_0)) \cdot f'(x_0) .$$

$$E_s: \quad f(x) = \sin x \quad f'(x) = \cos x$$

$$g(y) = e^y \quad g'(y) = e^y$$

$$(g \circ f)(x) = g(f(x)) = g(\sin x) = \\ \sin x \\ = e$$

$$(g \circ f)'(x) = g'(f(x)) \cdot f'(x) =$$

$$= e^{f(x)} \cdot \cos x = e^{\sin x} \cdot \cos x$$

$$D(e^{\sin x}) = e^{\sin x} \cdot \cos x$$

$$D(x) = 1$$

$$\begin{aligned} D(x^2) &= D(x \cdot x) = x \cdot 1 + 1 \cdot x \\ &= 2x \end{aligned}$$

$$\begin{aligned} D(x^3) &= D(x^2 \cdot x) = \\ &= 2x \cdot x + x^2 \cdot 1 = 3x^2 \end{aligned}$$

$$D(x^n) = nx^{n-1} \quad \forall n \in \mathbb{N}. \\ n \geq 1.$$

$$D(x^\alpha) \quad \alpha \in \mathbb{R}. \\ \textcircled{x > 0}$$

$$D(x^\alpha) = D(e^{\log(x^\alpha)}) =$$

$$= D(e^{\alpha \log x}) =$$

composizione

derivata di $\alpha \log x$

$$= e^{\alpha \log x} \cdot \left(\alpha \cdot \frac{1}{x} \right)$$

$$= (e^{\log x})^{\alpha} \cdot \frac{\alpha}{x} = x^{\alpha} \cdot \frac{\alpha}{x} =$$

$$= \boxed{\alpha x^{\alpha-1}}$$

$$E_{-} \text{ s.t. } \alpha = \frac{1}{2} .$$

$$x^{1/2} = \sqrt{x}$$

$$D(x^{1/2}) = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}} .$$

$$\underline{Es}: f(x) = a^x \quad a > 0$$

$$f'(x) = ?$$

$$a^x = e^{\log(a^x)} = e^{x \cdot \log a}$$

$$f'(x) = e^{x \cdot \log a} \cdot \log a =$$

$$= a^x \cdot \log a$$

Es: funzione continua senza
derivata in un punto.

$$f(x) = \begin{cases} x \sin \frac{1}{x} & \text{se } x \neq 0 \\ 0 & \text{se } x = 0 \end{cases}$$

f è continua in 0? *limitata*

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0 = f(0)$$

f è continua in $x_0 = 0$.

È derivabile?

$$\lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0} =$$

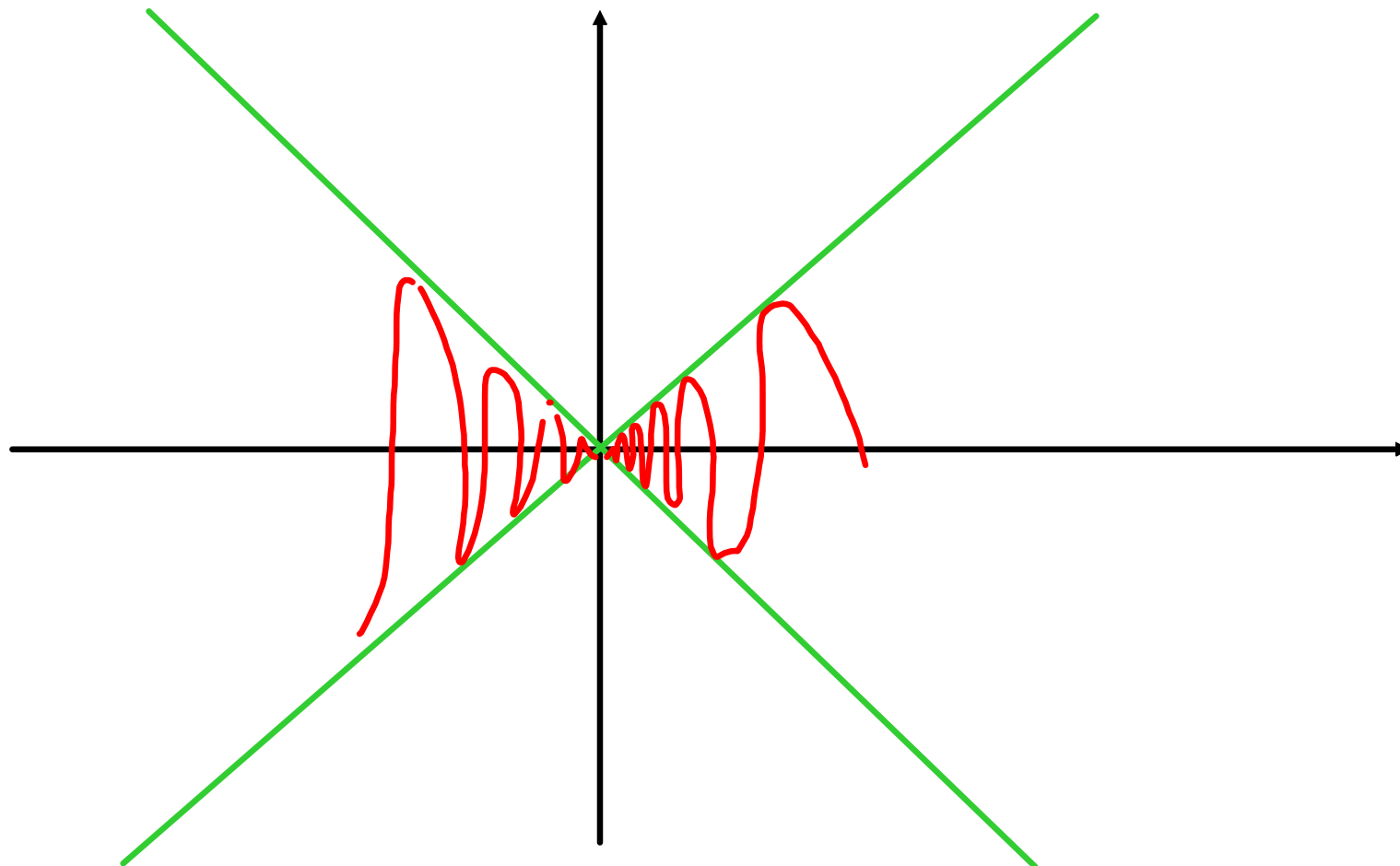
$$= \lim_{x \rightarrow 0} \frac{x \sin \frac{1}{x} - 0}{x} = \lim_{x \rightarrow 0} \sin \frac{1}{x}$$

non esiste

$$\lim_{x \rightarrow 0^+} \sin \frac{1}{x} = \lim_{t \rightarrow +\infty} \sin t \leftarrow \nexists$$

$$\frac{1}{x} = t \quad \text{se } x \rightarrow 0^+ \Rightarrow t \rightarrow +\infty.$$

quindi f non ha derivata
in $x_0 = 0$.



$$-x \leq x \sin \frac{1}{x} \leq x$$

$$(x > 0)$$

$$\underline{E}_5: \boxed{D \arctg}$$

$$f(x) = \operatorname{tg} x \quad f'(x) = 1 + (\operatorname{tg} x)^2$$

$$f^{-1}(y) = \arctg y, \quad y = \operatorname{tg} x$$

$$\left(f^{-1}(y)\right)' = \frac{1}{f'(f^{-1}(y))} =$$

$$= \frac{1}{1 + \left[\operatorname{tg} (f^{-1}(\eta)) \right]^2} =$$

$$= \frac{1}{1 + \left(\operatorname{tg} (\operatorname{arctg} y) \right)^2} = \frac{1}{1 + y^2}$$

$$E_s: f(x) = (1+x)^\alpha \quad \alpha \in \mathbb{R}$$

$$x > -1$$

$$f'(x) = \alpha (1+x)^{\alpha-1}$$

$$f(0) = 1, \quad f'(0) = \alpha$$

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + o(x-x_0)$$

$$x_0 = 0 \quad f(x) = (1+x)^\alpha$$

$$(1+x)^\alpha = 1 + \alpha x + o(x) \quad \text{per } x \rightarrow 0$$

$$\underline{E}_3: \quad \sqrt{1+x} = (1+x)^{1/2} =$$

$$= 1 + \frac{x}{2} + o(x) \quad \alpha = \frac{1}{2}$$

$${}^3\sqrt{1+x} = (1+x)^{1/3} \quad \alpha = \frac{1}{3}$$

$$= 1 + \frac{x}{3} + o(x)$$

$$\underline{E}_s: \lim_{x \rightarrow \infty} \left[\sqrt[3]{x^2 + 8x} - \sqrt[3]{x^2} \right] x^{1/3}$$

$$\left[(x^2 + 8x)^{1/3} - x^{2/3} \right] x^{1/3} =$$

$$= \left[\left[x^2 \left(1 + \frac{8}{x} \right) \right]^{1/3} - x^{2/3} \right] x^{1/3} =$$

$$= \left[x^{2/3} \left(1 + \frac{8}{x} \right)^{1/3} - x^{2/3} \right] x^{1/3} =$$

$$= x^1 \left(\left(1 + \frac{8}{x} \right)^{1/3} - 1 \right) =$$

$$(1+t)^\alpha = 1 + \alpha t + o(t)$$

$$\alpha = \frac{1}{3} \quad t = \frac{8}{x}$$

$$= x \left(\cancel{1} + \frac{1}{3} \frac{8}{x} + o\left(\frac{1}{x}\right) - \cancel{1} \right) \rightarrow \frac{8}{3}$$