NOTE ES. 25-27 OTT. 19

Note Title 9/27/2019

GRIS. 1

DOM 1

$$x^2 - \frac{1}{x} < 0$$

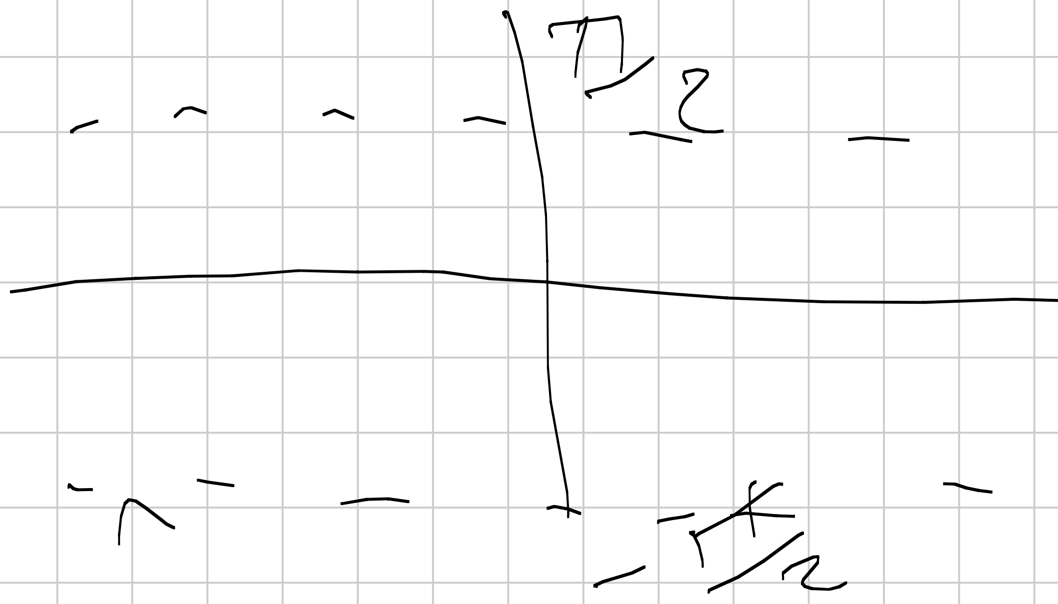
- necc. $x \neq 0$ quindi $x^2 > 0$
  per cui se $x < 0$ anche $\frac{1}{x} < 0$
  e quindi $x^2 - \frac{1}{x} > 0$
- Per cui le soluzioni sono tutte > 0
- se $x > 0$ $\quad x^2 - \frac{1}{x} < 0 \iff \underline{x^3 - 1 < 0}$
  x è sol. $\iff \quad 0 < x < 1$

D. 2 $f(x) = \operatorname{arctg} x + e^{\operatorname{arctg} x}$

$f(x) = F(g(x))$

$g(x) = \operatorname{arctg} x$

$F(y) = y + e^{y}$



$-\frac{\pi}{2} + \frac{1}{e^{\frac{\pi}{2}}} < f(x) \leq \frac{\pi}{2} + e^{\frac{\pi}{2}}$

D3 $\quad n \in \mathbb{N} \quad \log\left(1 + \frac{(-1)^n}{2n}\right) < 0$

$n \geq 1$

$n \neq 0$

$$\log \boxed{} < 0 \iff 0 < \boxed{} < 1$$

$n \geq 1 \qquad \cancel{1} + \frac{(-1)^n}{2n} < \cancel{1}, \quad \frac{(-1)^n}{2n} < 0,$

$(-1)^n < 0 \iff n = 2m+1 \quad m \in \mathbb{N}$

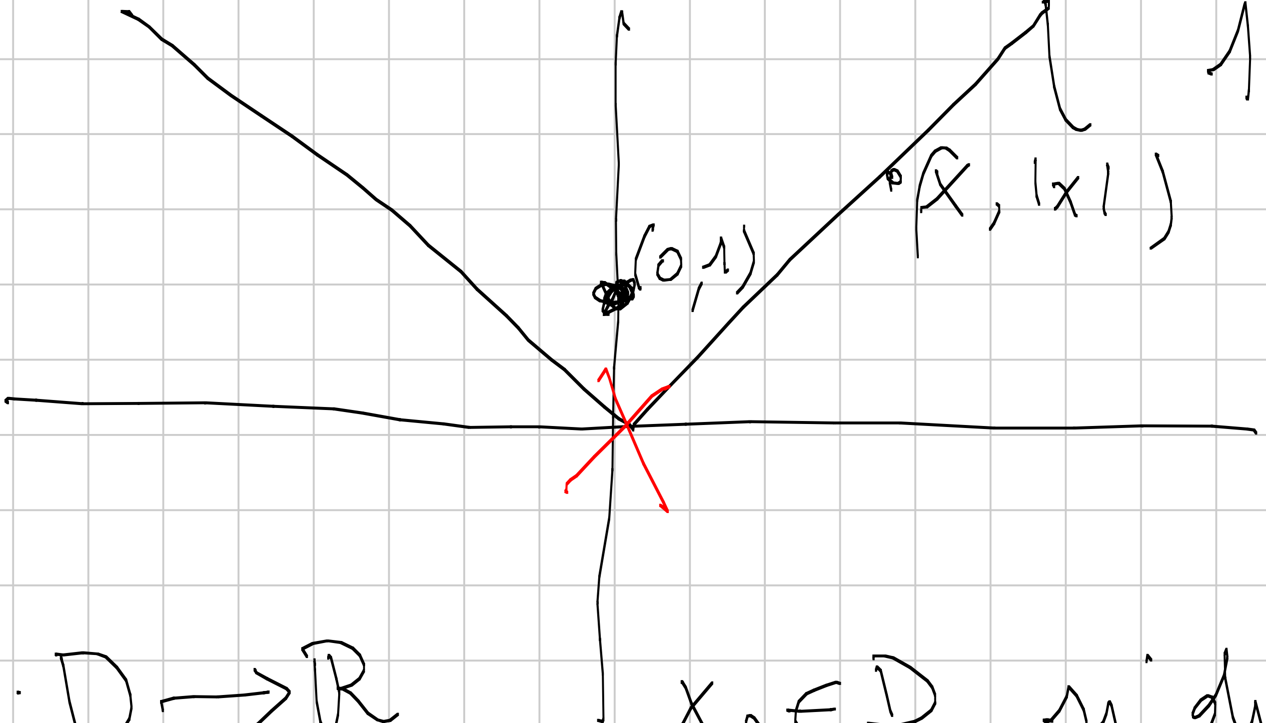
D. 4 $\quad \sin(x^2) = 1$ se $x = \sqrt{\frac{\pi}{2} + 2n\pi}$

D.5 $\quad \sin^2 x = (\sin x)^2 \qquad n \in \mathbb{N}$

$\uparrow$

esercizi lasciati: disegnare a mano i grafici $\sin(2x)$, $\sin(4x)$, $\sin(x^2)$, $(\sin x)^2$

D 6 $\quad f(x) = \begin{cases} |x| & x \neq 0 \\ 1 & x = 0 \end{cases}$



Im f $\quad \cancel{0}$

$f : D \to \mathbb{R}$, $x_0 \in D$ si dice massimo locale (relativo) se $\exists\, r > 0$

$\forall x \in D : dist(x, x_0) < r \quad f(x) \leq f(x_0)$

CIOÈ $x_0 \in D$ è di massimo locale per $f: D \to \mathbb{R}$

se $\exists\, r > 0$ per cui x_0 è di massimo (assoluto) per $f: D \cap (x_0 - r; x_0 + r) \to \mathbb{R}$

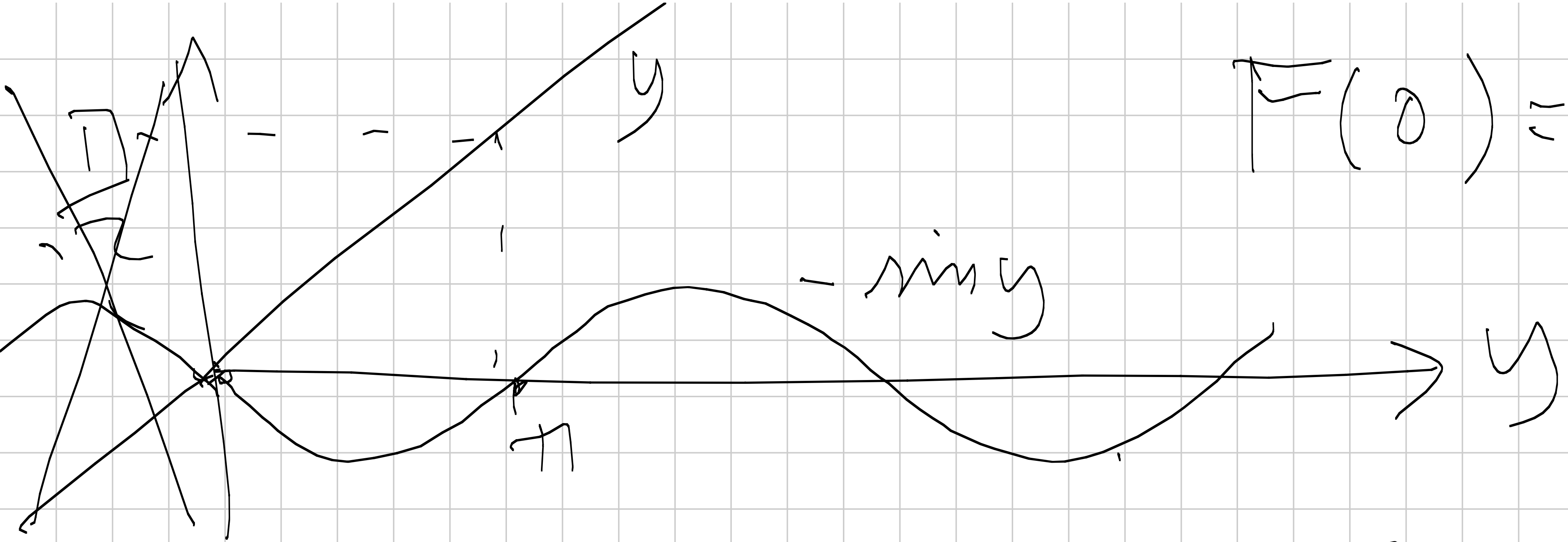
D7 $f(x) = |x| + \sin|x| =$

$= F(|x|)$

$F(y) = y + \sin y \quad \boxed{y \geq 0}$

$f(x) \geq |x| - 1 \Rightarrow$ ~~non A~~ non D

? $y \geq 0 \Rightarrow y + \sin y \geq 0 \quad (y \geq -\sin y)$



$F(0) = 0$

we $y \geq \pi$

$F(y) \geq 0$

$y \geq \pi \geq 1 \geq -\sin y$

si

$0 \leq y \leq \pi \quad -\sin y \leq 0 \longrightarrow F(y) \geq 0$

foglio di ri,m delle forma stt.

D 5

$$[f(x)]^{g(x)} = e^{\lg f^g} = \boxed{e^{g(x)\lg f(x)}}$$

$$x^{\lg x} = e^{\lg x \lg x} = e^{(\lg x)^2}$$

D 7

$$\begin{cases} \sin x > \frac{\sqrt{3}}{2} \\ -6 < x < 0 \end{cases}$$

$2\pi > 6{,}28 > 6$

$6 > \frac{5}{3}\pi$

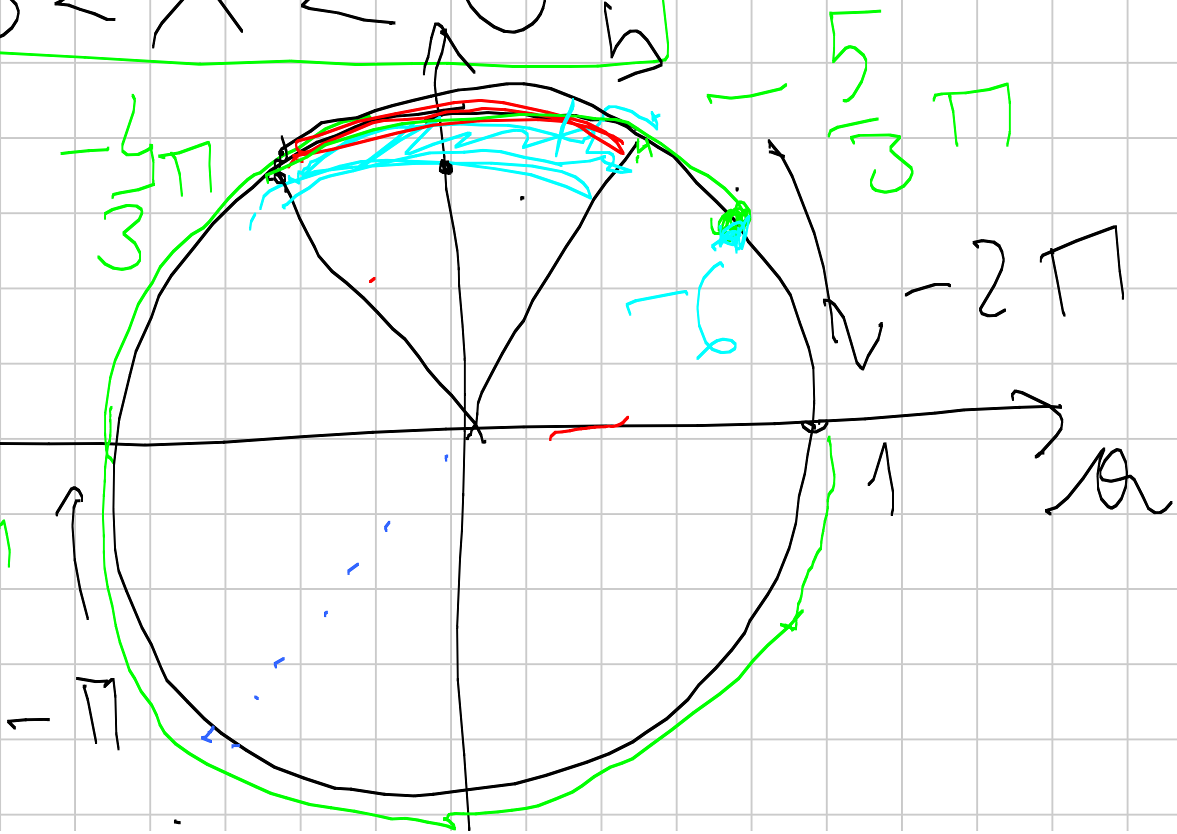
$18 > 5\pi$

$2k\pi + \frac{\pi}{3} < x < 2k\pi + \frac{2}{3}\pi$

$6 \quad \frac{4}{3}\pi$

$18 > 4 \cdot 3{,}2 = 12{,}8 > 4\pi$

$-6 < -\frac{4}{3}\pi$



D 8

$$A = \left\{ 9,\ \underset{9{,}9}{9 + \frac{9}{10}},\ \underset{9{,}99}{9 + \frac{9}{10} + \frac{9}{100}},\ \underset{9{,}999}{9 + \frac{9}{10} + \frac{9}{100} + \frac{9}{1000}},\ \ldots,\ \underset{9{,}\underbrace{9\ldots9}_{n}}{\left(9 + \frac{9}{10} + \ldots + \frac{9}{10^{n+1}}\right)};\ \ldots \right\}$$

$$A \leq 10 \qquad 9{,}\overline{9} = 10$$

$$1 - x^{n+1} = (1 - x)(1 + x + x^2 + \dots + x^n)$$

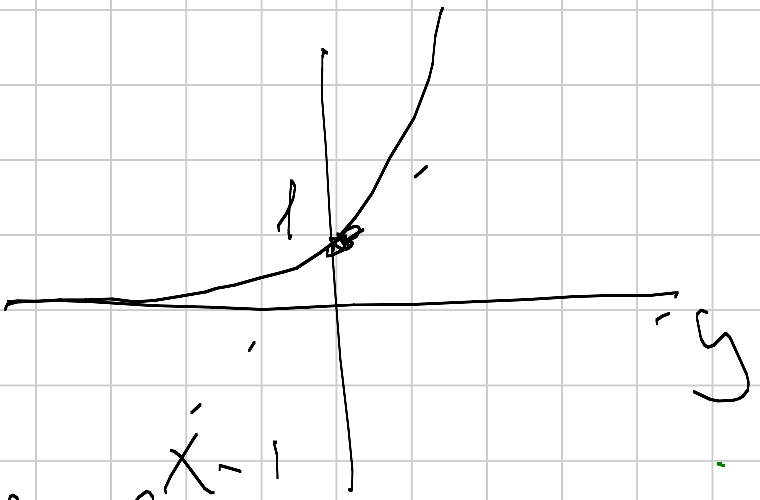
$$\frac{1 - x^{n+1}}{1 - x} = 1 + x + x^2 + \dots + x^n$$

$$x = \frac{1}{10} \qquad 1,\underbrace{1\,1 \dots 1}_{n}$$

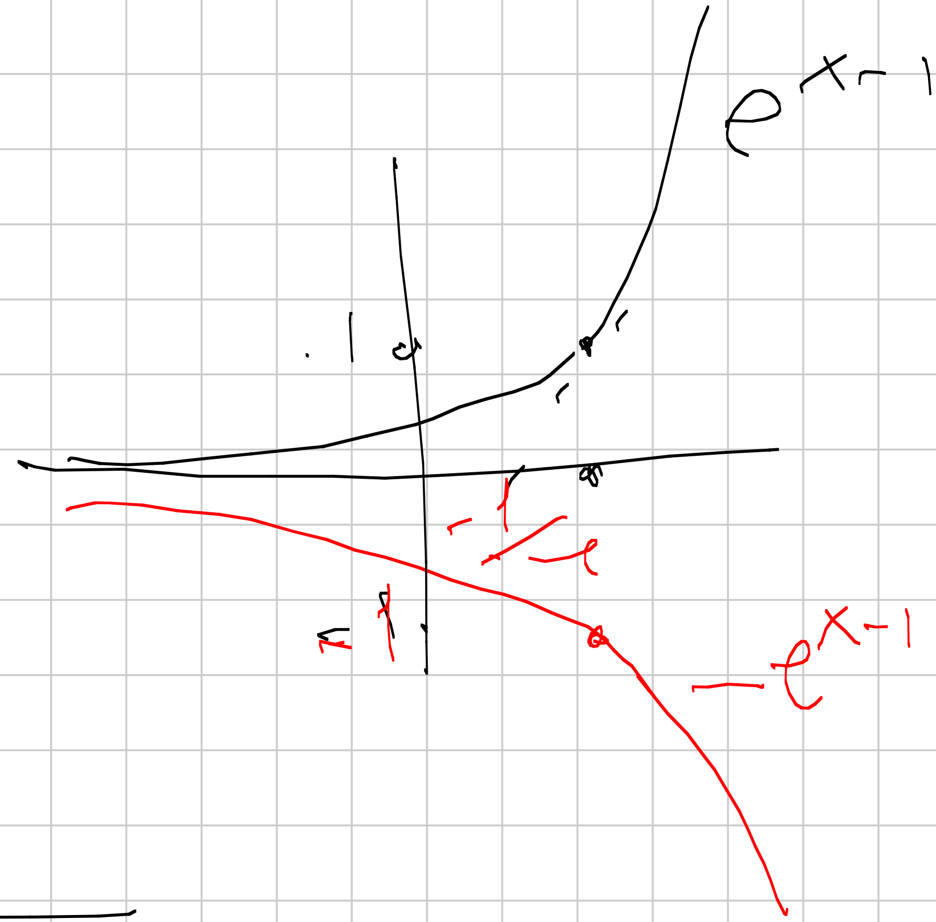
$$\frac{1}{1 - \frac{1}{10}} \qquad \frac{1 - \frac{1}{10^{n+1}}}{1 - \frac{1}{10}} = 1,1\dots1 \qquad 1,\overline{1}$$

$$\frac{10}{9} = 1,\overline{1}$$

$D \quad 1 - e^{x-1})$



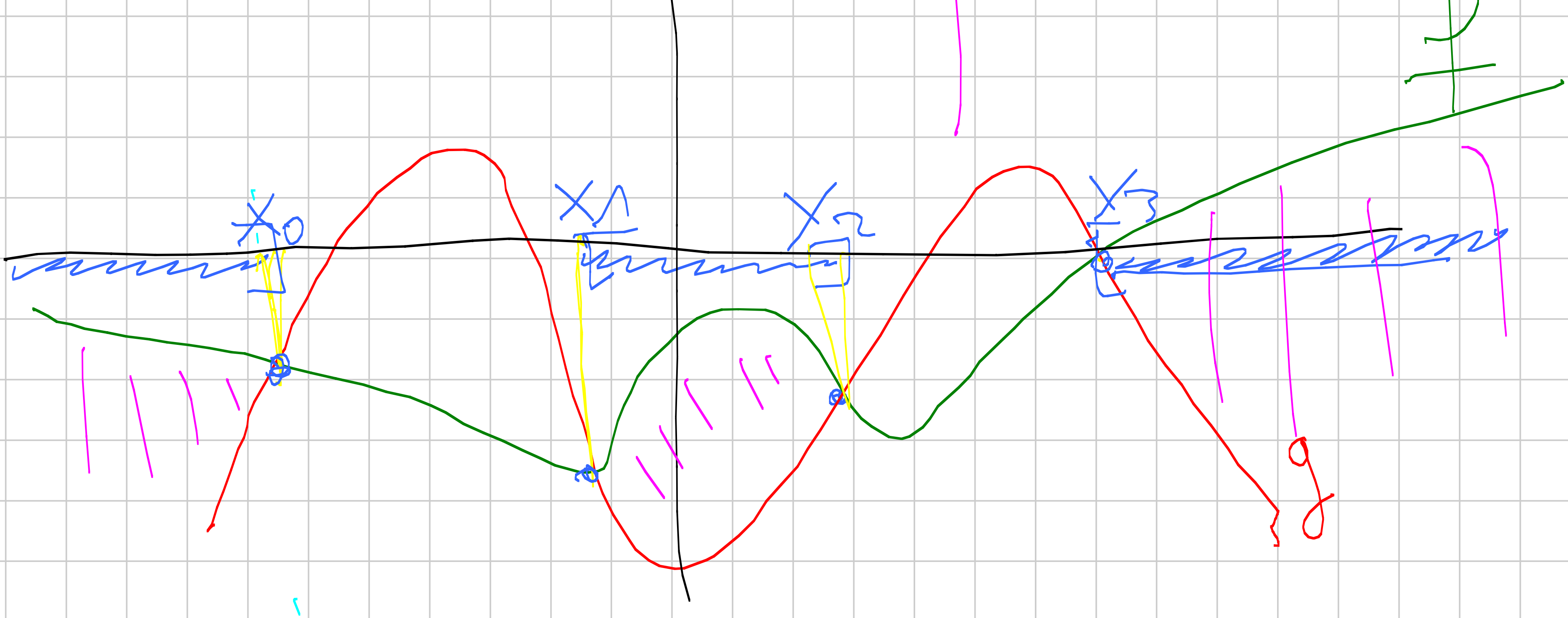
$e^{x-1} \longrightarrow y = x - 1$



$1 - e^{x-1} \longrightarrow$

D10 $x : f(x) \geq g(x)$

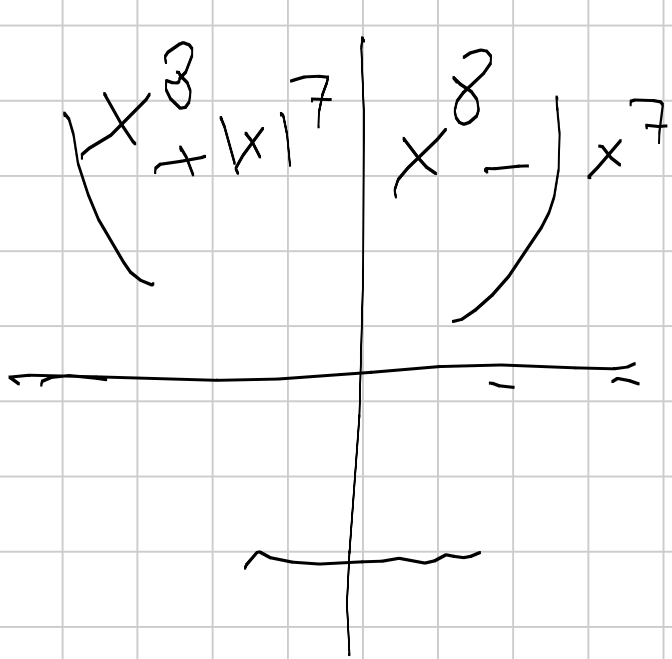
è diverso da disegnare $\{(x, y) : f(x) \geq y \geq g(x)\}$



PRIMO FOGLIO TESTI II sett.

D. 1

$$\sum_{0 \le n \le 8} (-1)^n x^n \quad \ldots \quad \sim$$



D. 2 $\quad e^{\sin x + 3x\cos x}$

e^{y} è ing

$\sin x + 3x\cos x$

$x = 2k\pi \longrightarrow 6k\pi$ illimitat

$x = \frac{\pi}{2} + 2k\pi \longrightarrow 1$

$f\left(\frac{\pi}{2} + 2k\pi\right) = e$

$$x = -\frac{\pi}{2} + 2k\pi \qquad k \geq 1$$

$$f\left(-\frac{\pi}{2} + 2k\pi\right) = e^{-2k\pi + \frac{\pi}{2}}$$

$$= \left(e^{2\pi}\right)^{-k} \cdot e^{\frac{\pi}{2}} = \frac{1}{\left(e^{2\pi}\right)^{k}} e^{\frac{\pi}{2}}$$

$$\operatorname{Im} f \subseteq (0; +\infty)$$

$$\operatorname{Im} f \supseteq \left\{ f\left(2k\pi - \frac{\pi}{2}\right) : k \geq 1 \right\} =: K$$

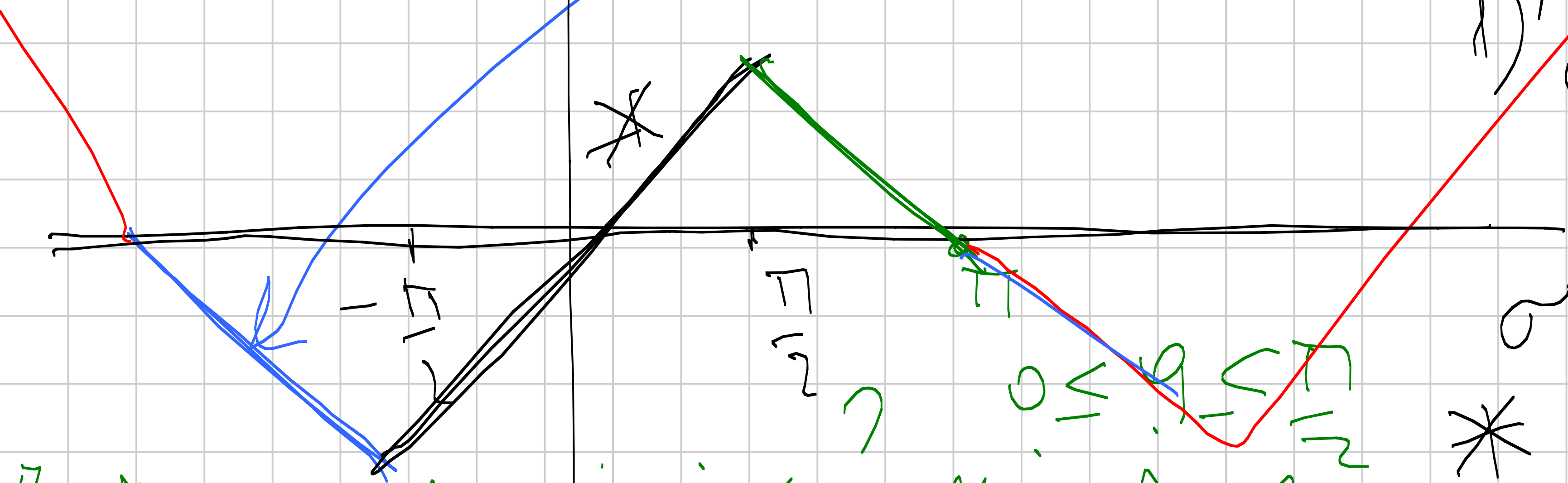
$$\inf K = 0 \Rightarrow \inf \operatorname{Im} f = 0$$

D 8 graf $\arcsin(\sin x)$

2) $\arcsin(\sin x)$ è 2π-per.

3) è dispari: basta $[0, \pi]$ $x \in \mathbb{R}$

1) per def $\arcsin$ è l'inversa di $\sigma = \sin|_{[-\frac{\pi}{2}, \frac{\pi}{2}]}$



$*$ $\arcsin \sigma(x) = x$

$\frac{\pi}{2} \leq x \leq \pi$ $\arcsin \sin x = \arcsin \sin y = y$

$y = \pi - x$ $0 \leq y \leq \frac{\pi}{2}$