NOTA ES del 2-4 ottobre 19

Note Title 10/2/2019

$$3 + \frac{1}{10} + \frac{4}{100} + \dots + \frac{P_n}{10^n}$$

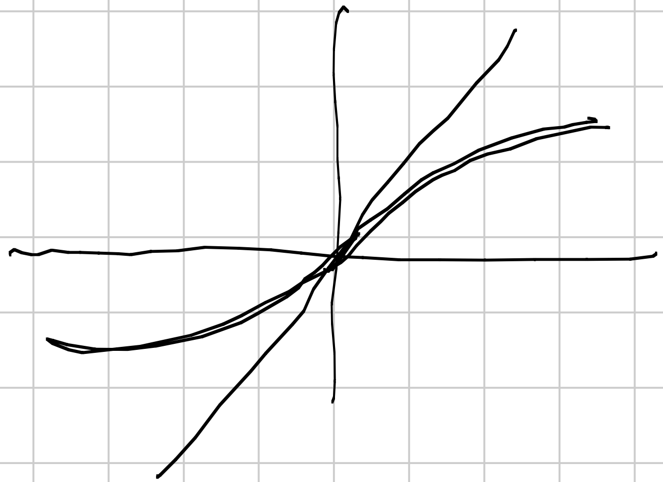
$$\pi = 3{,}14 \cdots P_n \dots$$

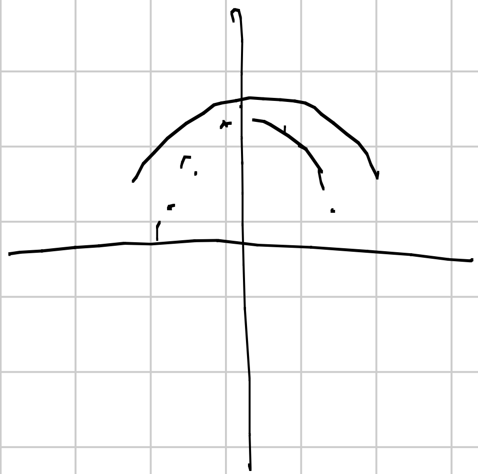
$$\pi = \sup \Big\{ \underbrace{3 + \frac{1}{10} + \dots + \frac{P_n}{10^n}}_{3{,}14 \cdots P_n} : n \in \mathbb{N} \Big\}$$

LIMITI NOTEVOLI TRIGONOMETRICI

$\exists \lim_{x \to 0} \frac{\sin x}{x} = 1$, $\exists \lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$

$\lim_{x \to 0} \frac{1-\cos x}{\frac{x^2}{2}} = 1$

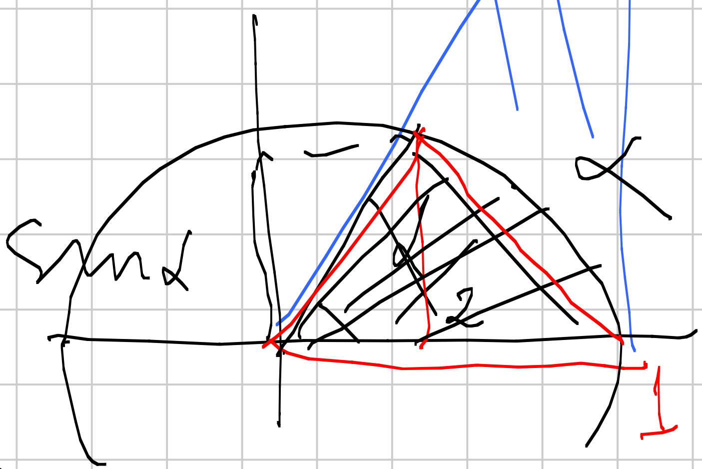




Ingredienti:

1) $|\sin\alpha - \sin\beta| \leq |\alpha - \beta|$

2) $|\sin\alpha| \leq |\alpha| \longrightarrow$ senza perdere generalità suppongo $0 \leq \alpha \leq \frac{\pi}{2}$ poiché il seno è dispari e $\frac{\pi}{2} \geq 1 \geq |\sin\alpha|$



α

$\geq$

$\frac{\sin\alpha}{2}$

TRIANGOLO $\subseteq$ SETTORE $\subseteq$ TRIANGOLONE

3) per $|\alpha| \leq \frac{\pi}{2}$ si ha $|\alpha| \leq |\operatorname{tg}\alpha|$

$$|\sin\alpha - \sin\beta| \leq |\alpha - \beta|$$

$$\sin a + \sin b = 2\sin\frac{a+b}{2}\cdot\cos\frac{a-b}{2}$$ PROSTAFERESI

$a = \alpha$

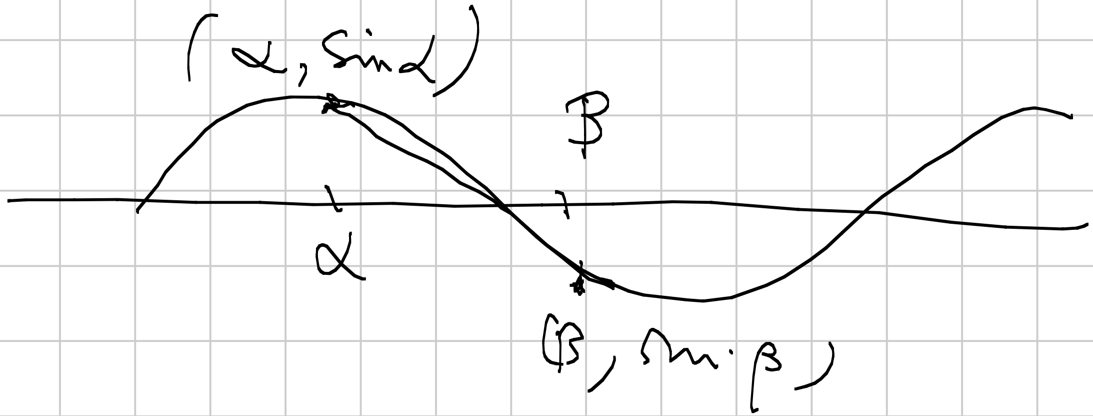
$-\beta = b$

$$\sin\alpha - \sin\beta = \sin\alpha + \sin(-\beta) = 2\sin\frac{\alpha-\beta}{2}\cdot\cos\frac{\alpha+\beta}{2}$$

$$|\sin\alpha - \sin\beta| = 2\left|\sin\frac{\alpha-\beta}{2}\right|\left|\cos\frac{\alpha+\beta}{2}\right| \leq 2\left|\sin\frac{\alpha-\beta}{2}\right|$$

$$\leq 2\left|\frac{\alpha-\beta}{2}\right| = |\alpha-\beta|$$

$$\left|\frac{\sin\alpha - \sin\beta}{\alpha - \beta}\right|$$



4) Conseguenza il seno è continuo

$$0 \le |\sin x - \sin x_0| \le |x - x_0|$$

$$\underset{0}{\downarrow} \qquad\qquad \underset{0}{\downarrow}\ x \to x_0$$

$$\cos x = \sin\left(x + \frac{\pi}{2}\right)$$

$$0 \le |\cos x - \cos x_0| \le |x - x_0|$$

DIMOSTRIAMO $\displaystyle \frac{\sin x}{x} \xrightarrow[x \to 0]{} 0$

$\sin x \xrightarrow[x \to 0]{} 0$

appena provato

$$|\sin x| \leq |x| \leq |\operatorname{tg} x|, \quad |x| < \frac{\pi}{2}$$

$\left(\lim\limits_{x \to x_0} f(x) \qquad \nexists \lim\limits_{x \to +\infty} \sin x \right)$

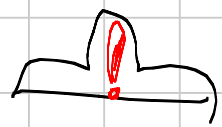
si assume per assurdo, poiché $x \to 0$ si assume anche

$0 \leq \cancel{x}$ $\qquad 0 \leq x < \frac{\pi}{2}$

$$\cos 0 \xleftarrow[x \to 0]{} \cos x \leq \frac{\sin x}{x} \leq 1 \xrightarrow[x \to 0]{} 1$$

$\cos 0 = 1$

CARAB.



SIMULAZIONE III settimana

D 1 $\quad f(x) = \begin{cases} x^{a} + a & x > 0 \\ a \cdot e^{ax} & x \leq 0 \end{cases}$ $\qquad a \in \mathbb{R}$

CONT. in $x = 0$ $\therefore$ $f(0) = a$

$\exists \lim_{x \to 0} f(x) = a = f(0)$

Devo vedere per quali a

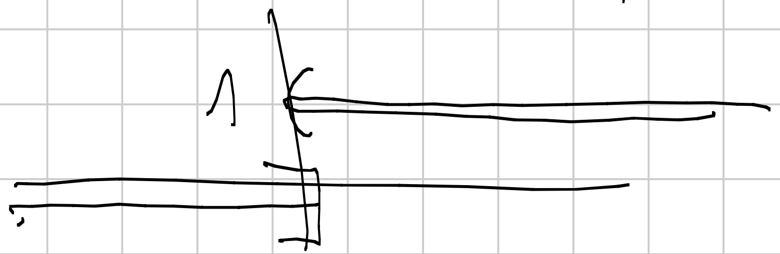
1) Esiste il limite destro

2) " " sinistro

3) sono eguali

3bis) sono eguali ad a

Se $a = 0$ $\quad f(x) = \begin{cases} x^0 + 0 & x > 0 \\ 0 & x \leq 0 \end{cases} = \begin{cases} 1 & x > 0 \\ 0 & x \leq 0 \end{cases}$



Se $a < 0$ $\quad f(x) = \begin{cases} x^{a} + a & x > 0 \\ a e^{ax} & x \leq 0 \end{cases}$

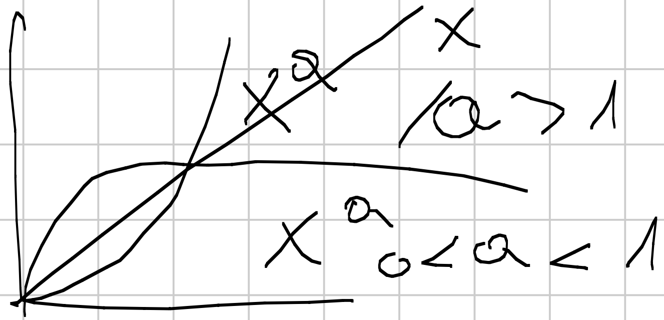
$$\lim_{x \to 0} a e^{ax} = a \cdot \lim_{x \to 0} e^{ax} \underset{\substack{\uparrow \\ \text{CONTINUITÀ}}}{=} a e^{a \cdot 0} = a$$

$$\| $$

$$\lim_{x \to 0^-} f(x)$$

$$\lim_{x \to 0^+} f(x) \overset{?}{=} \lim_{x \to 0^+} (x^{a} + a) = a + \lim_{x \to 0^+} x^{a} =$$

$$= a + \lim_{y \to +\infty} y^{|a|} = +\infty \qquad y = \frac{1}{x}$$



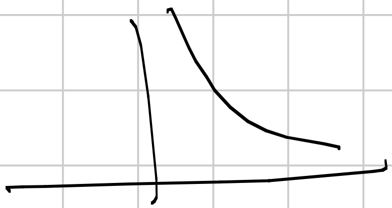
x^a $a<0$

$a > 0$

$$\lim_{0^-} f = a$$

$$\lim_{x \to 0^+} f = \ell + \lim_{x \to 0^+} x^a = \ell$$

$a < 0$



$$x^a \xrightarrow[x \to 0^+]{} +\infty$$

$a > 0$

$$x^a \xrightarrow[x \to 0^+]{} 0$$

D3

arcsin è continua.

quindi basta mostrare

$$\nexists \lim_{x \to +\infty} \cos\left(\pi[x] + \frac{\pi}{x}\right) \in [-1,1]$$

$x = 2n \quad n \in \mathbb{N} \qquad 2\pi n$

$$\cos\left(\pi[x] + \frac{\pi}{x}\right) =$$

$$= \cos(\pi[x]) \cdot \cos\frac{\pi}{x} - \cancel{\sin(\pi[x])\sin\frac{\pi}{x}}$$

$$= (-1)^{[x]} \cos\frac{\pi}{x} = \begin{cases} \cos\frac{\pi}{x} & [x] \text{ pari} \\ -\cos\frac{\pi}{x} & [x] \text{ dispari} \end{cases}$$

$$\lim_{x \to +\infty} \frac{1}{x} = 0$$

$$\forall \varepsilon > 0 \quad \exists r_{>0} \; \forall x > r \quad 0 \le \frac{1}{x} \le \varepsilon$$

$$x \in (r, +\infty)$$

prendo $r = \frac{1}{\varepsilon}$

$$\cos\left(\pi\left([x]+\frac{1}{x}\right)\right) = \begin{cases} \cos\frac{\pi}{x} & [x] \text{ pari} \\ -\cos\frac{\pi}{x} & [x] \text{ dispari} \end{cases}$$

per cont.
di coseno in 0

$x \to +\infty$, $\frac{\pi}{x} \to 0^+$

$$\begin{cases} \longrightarrow 1 \\ \longrightarrow -1 \end{cases}$$

D 3

$$\frac{x^{11} - 3x^2 + \sin x}{1 - \cos x} \qquad x \to 0$$

$$\frac{x^{11} - 3x^2 + \sin x}{\frac{x^2}{2}} \cdot \frac{\frac{x^2}{2}}{1 - \cos x} \xrightarrow[x \to 0]{} 1$$

$$\frac{x^{11} - 3x^2 + \sin x}{\frac{x^2}{2}}$$

$$\frac{\frac{x^2}{2}}{1 - \cos x}$$

$$\left[2x^9 - 6 + 2\,\frac{\sin x}{x^2}\right] \cdot \frac{\frac{x^2}{2}}{1 - \cos x}$$

$$2x^9 \to 0 \qquad -6 \to 6 \qquad \frac{\sin x}{x} \cdot \frac{1}{\cancel{x}} \qquad \frac{\frac{x^2}{2}}{1-\cos x} \to 1$$

D. L

$$x \to +\infty$$

$$\lim_{x \to +\infty} \frac{\sqrt[3]{x^2+8x} - \sqrt[3]{x^2}}{\sin\left(x^{-1/3}\right)}$$

$$\frac{(x^2+8x)^{1/3} - x^{2/3}}{\sin\left[\left(\frac{1}{x}\right)^{1/3}\right]}$$

$$\frac{(x^2+8x)^{1/3} - (x^2)^{\frac{1}{3}}}{\left(\frac{1}{x}\right)^{\frac{1}{3}}} \qquad \frac{\left(\frac{1}{x}\right)^{\frac{1}{3}}}{\sin\left[\left(\frac{1}{x}\right)^{\frac{1}{3}}\right]}$$

$$x \to +\infty$$

$$\downarrow \qquad 1 \qquad \left(\frac{1}{x}\right)^{\frac{1}{3}} \to 0$$

$$a^3 - b^3 = (a-b)(a^2+ab+b^2)$$

$$\frac{a^3-b^3}{a^2+ab+b^2} = a-b \qquad a = \sqrt[3]{x^2+8x} \quad b = \sqrt[3]{x^2}$$

$$\frac{8x}{(x^2+8x)^{2/3} + x^{4/3} + (x^4+8x^3)^{1/3}} \to C$$

$$\downarrow 1$$

$$\frac{8x}{(x^2+8x)^{2/3}+x^{4/3}+(x^4+8x^3)^{1/3}}$$

$$\frac{\frac{1}{\sqrt[3]{x}}}{\frac{1}{\sqrt[3]{x}}} \to 1$$

$x \to +\infty$

$x > 1 \quad a > b > 0 \Rightarrow x^a > x^b$

$$\boxed{\lim_{x\to+\infty} \frac{x^a}{x^b} = \lim_{x\to+\infty} x^{a-b} = +\infty} \qquad a-b>0$$

$$\frac{8x}{x^{4/3}\left[\left(1+8/x\right)^{2/3}+1+\left(1+8/x\right)^{4/3}\right]} \cdot \quad \to 1$$

$$\frac{x}{x^{4/3}} = \frac{1}{x^{1/3}} \longrightarrow 0$$

D5

$$\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{2x - \pi}$$

$x \to \frac{\pi}{2}$

$$\boxed{x - \frac{\pi}{2}} \to 0$$

$$\frac{\cos\left(x - \frac{\pi}{2} + \frac{\pi}{2}\right)}{2\left(x - \frac{\pi}{2}\right.}$$

$x - \frac{\pi}{2} \to 0$

$\frac{\sin \square}{\square} \quad \square \to 0 \quad \to 0$

$$\frac{\cos\left(\left(x - \frac{\pi}{2}\right) + \frac{\pi}{2}\right)}{2\left[x - \frac{\pi}{2}\right]}$$

$$-\frac{\sin\left(x - \frac{\pi}{2}\right)}{2\left[x - \frac{\pi}{2}\right]} \longrightarrow -\frac{1}{2}$$

D 7 $\lim\limits_{x \to 0} \dfrac{x \sin(3x)}{1 - \cos(6x)}$

$\cancel{x} \dfrac{\sin 3x}{3x} \, 3\cancel{x} \cdot \dfrac{\frac{36x^2}{2}}{1 - \cos(6x)} \dfrac{1}{\frac{36}{2} \cancel{x^2}} \longrightarrow \dfrac{1}{6}$

D 3

$$\lim_{x \to +\infty} f = 2$$

$$\frac{2x^2 - 3x}{x^2+1} = \frac{2x^2+2}{x^2+1} - \frac{2+3x}{x^2+1}$$

$$= 2 - \frac{2+3x}{x^2+1} < 2 \quad \forall x > 0$$

$2x^2 - 3x = x(2x-3) > 0$ per $x > \frac{3}{2}$

ove f è negativo è < 0 per $0 < x < \frac{3}{2}$

continua per W. c'è MINIMO

D10 $\quad \dfrac{\sin x}{[\sin(2x)]^2} \qquad 0 < x < \dfrac{\pi}{2}$

C.N.O. poiché $f \geq 0$

$$\frac{\sin x}{4x^2}\left(\frac{2x}{\sin 2x}\right)^2 = \underbrace{\frac{\sin x}{x}}_{\to 1} \cdot \underbrace{\frac{1}{4x}}_{\substack{\to +\infty \\ 0^+}} \cdot \underbrace{\left(\frac{2x}{\sin 2x}\right)^2}_{\to 1}$$