Note del 9/11 ottobre 2019

Note Title 10/9/2019

IV foglio: similazioni

D.1 fatta da Grisa.

D.2 $\dfrac{\sin x}{x} \to 1$ $\quad x \to 0$

$\dfrac{\sin x}{x} - 1 \to 0$ $\quad x \to 0$ $\quad = d(x)$

$\dfrac{\sin x}{x} = 1 + d(x)$

$\sin x = x + r(x)$

$r(x) = x\, d(x)$

$\sin x = x + o(x)$

$$\frac{1-\cos x}{\frac{x^2}{2}} \xrightarrow[x \to 0]{} 1 \qquad 1-\cos x = \frac{x^2}{2} + o(x^2)$$

$N - qD$

$$\frac{N}{D} \xrightarrow[x \to x_0]{} q \in \mathbb{R} \qquad \sim \quad N = qD + o(D)$$

$N \to 0$

$D \to 0$

$$\frac{N}{D} = q + \frac{o(D)}{D}$$

D 2 $\quad \dfrac{x^2 \sin(2x)}{\ln(1+(\sin 3x)^3)} \simeq \quad x \to 0$

$\dfrac{\ln(1+q)}{q} \to 1, \quad q \to 0$

$\simeq \dfrac{x^2 \cdot (2x + o(x))}{(\sin 3x)^3 + o(\sin 3x)^3}$

$\dfrac{x^2 o(x)}{x^3}$

$\simeq \dfrac{2x^3 + o(x^3)}{(3x + o(x))^3 + o(3x + o(x))^3}$

$\dfrac{(o(x))^2}{x^2}$

$$= \frac{2x^3 + o(x^3)}{27x^3 + \cancel{x^2 o(x)} + \cancel{x\,[o(x)]^2} + \cancel{o(x^3)} + o(x^3 + o(x^3))}$$

$o(x^3)$

$$= \frac{2x^3 + o(x^3)}{27x^3 + o(x^3)} = \frac{x^3\left(2 + \frac{o(x^3)}{x^3}\right)}{x^3\left(27 + \frac{o(x^3)}{x^3}\right)} \to 0$$

$$= \frac{2x^3}{27x^3} \cdot \frac{1 + \frac{o(1)}{1}}{1 + o(1)} \to 1$$

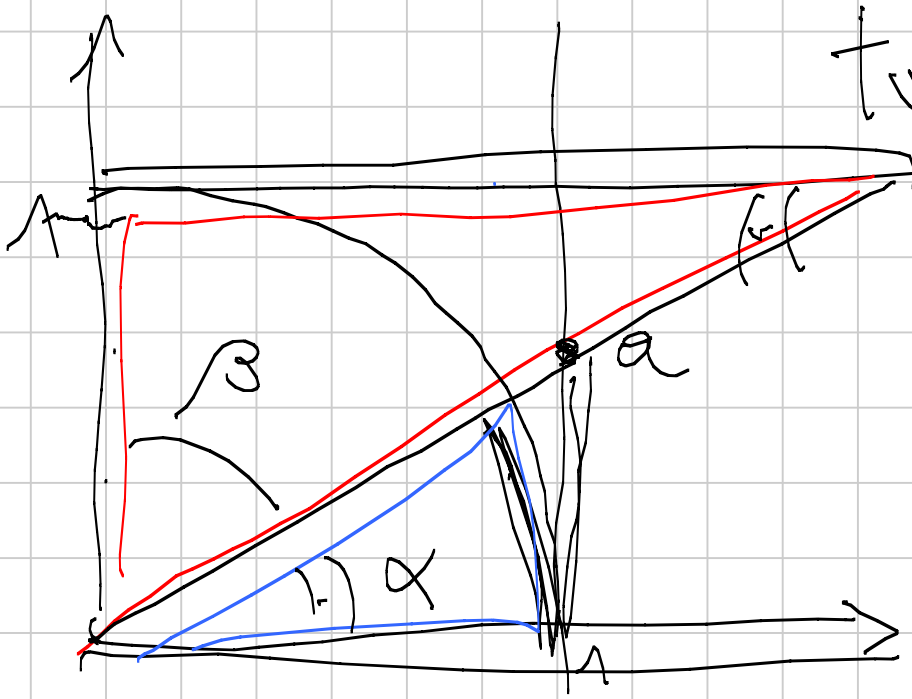
$$\frac{o(x^3 + o(x^3))}{x^3 + o(x^3)} \cdot \frac{x^3 + o(x^3)}{x^3} \to 0$$

$$\frac{x^2 \cdot \frac{\sin(2x)}{2x}}{\ln\left(1+(\sin 3x)^3\right)} \cdot \frac{2x}{27x^3}$$

$$\frac{1}{(\sin 3x)^3} \cdot \frac{(\sin 3x)^3}{(3x)^3} \cdot (3x)^3$$

D3 $\operatorname{arctg} a + \operatorname{arcctg} \frac{1}{a} = \frac{\pi}{2}$

$a > 0$



$\operatorname{tg} \beta$

$\alpha + \beta = \frac{\pi}{2}$

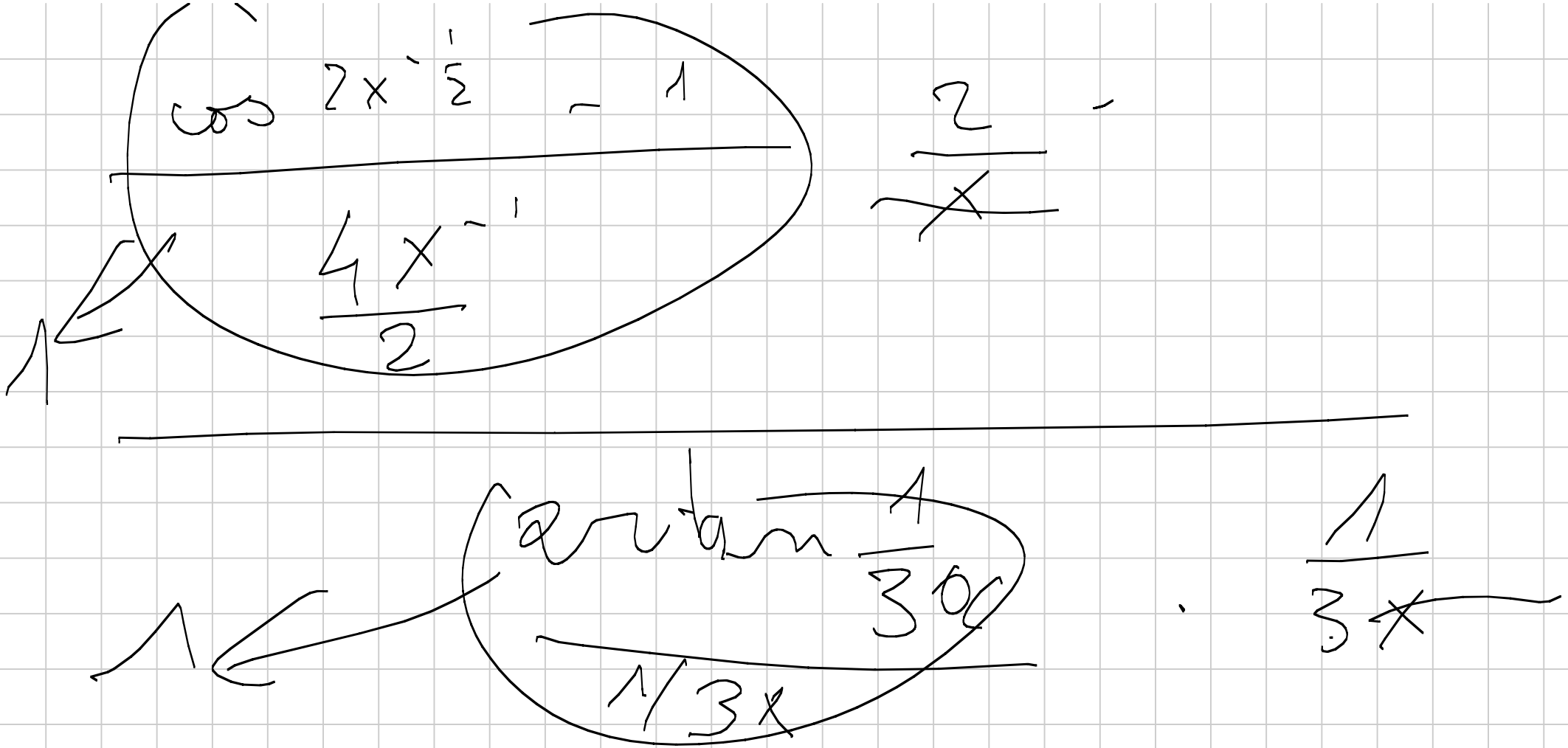
$a = \operatorname{tg} \alpha$

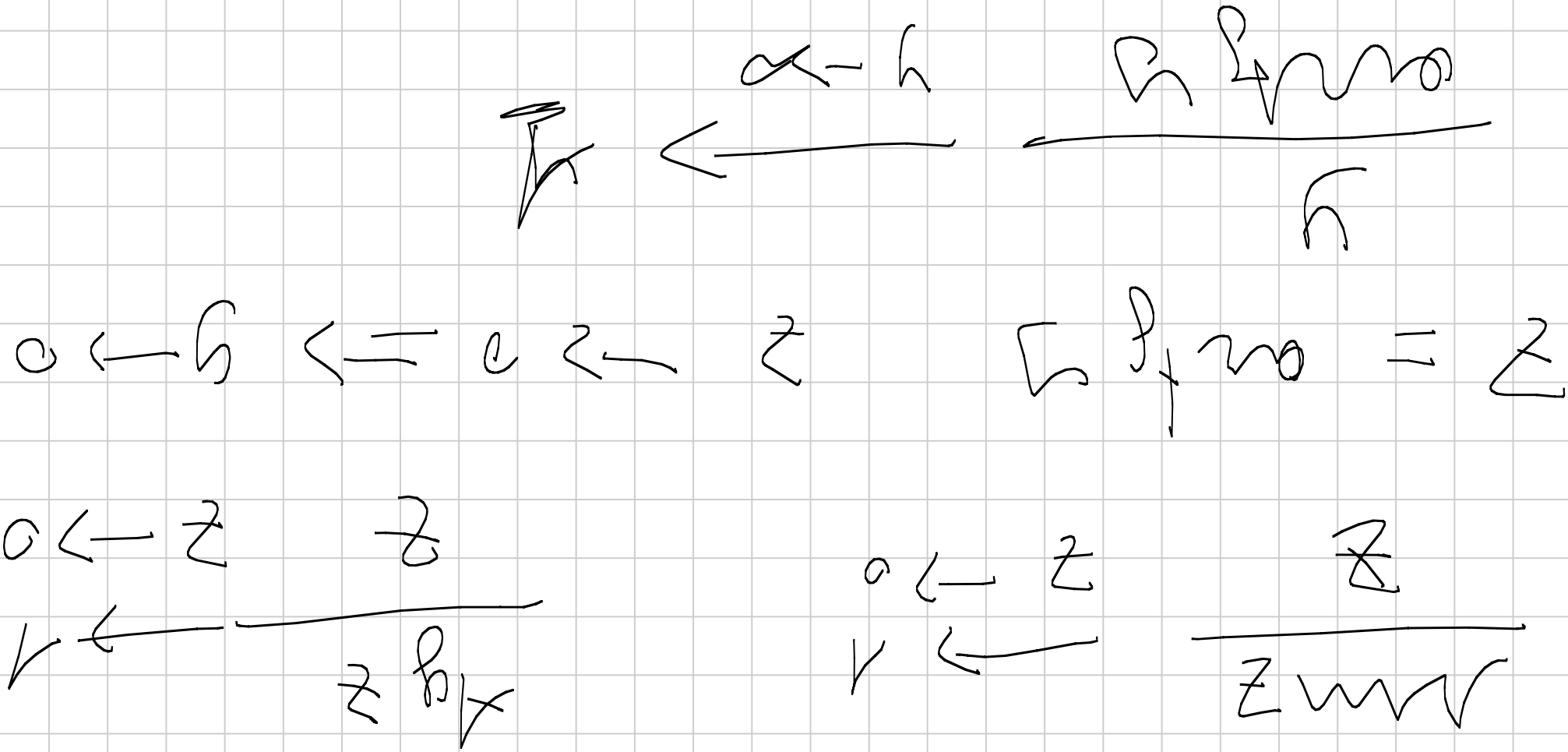
$\frac{\operatorname{tg} \beta}{1} = \frac{1}{a}$

D3 $\cos\frac{2}{\sqrt{x}} - 1 \to 0 \qquad x \to +\infty$

$\frac{\pi}{2} - \operatorname{arctg} 3x \to 0$

$$\frac{-\frac{1}{2}\left(\frac{2}{\sqrt{x}}\right)^2 + o\left(\frac{1}{x}\right)}{\operatorname{arctg}\frac{1}{3x}} = \frac{-\frac{1}{2}\cdot\frac{4}{x} + o\left(\frac{1}{x}\right)}{\frac{1}{3x} + o\left(\frac{1}{3x}\right)}$$





DL, $\dfrac{\sqrt{1+x^2}-1}{e^{(x^3)}-1}$ $x>0$

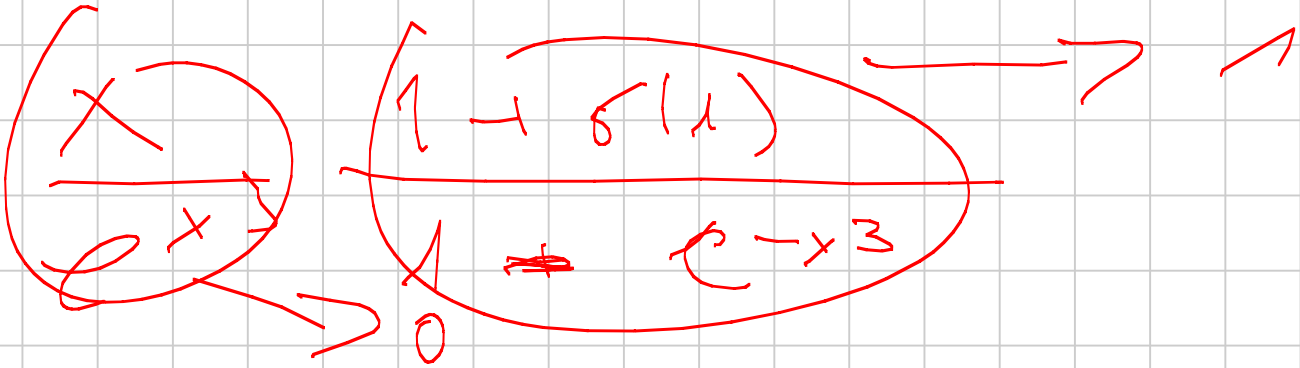
$\sqrt{1+x^2} = x + o(x)$ $x \to 0,0$

$\dfrac{\sqrt{1+x^2}}{x} = 1 + o(1)$

$\dfrac{x \cdot \sqrt{1+\frac{1}{x^2}}}{x}$ $x \to +\infty$, $\frac{1}{x^2} \to 0$

$\dfrac{x + o(x)}{e^{x^3}-1}$

$-1 = o(x)$ $x \to +\infty$



$$\frac{x}{e^x} \to 0 \qquad \frac{1+\sigma(x)}{1+e^{-x^3}} \to 1$$

$$\frac{\sqrt{1+x^2}-1}{e^{x^3}-1} > 0 \quad \forall x > 0$$

Ottimo è lim inferiore, ma non ha minimo

$$\begin{vmatrix} x_1 & x_2 & x_3 \\ 1 & 1 & 1 \\ x_1^2 & x_2^2 & x_3^2 \end{vmatrix} = (x_2 - x_1)(x_3 - x_1)(x_3 - x_2)$$

$$\begin{vmatrix} x_1 & x_2 & x_3 \\ 1 & 1 & 1 \\ x_1^2 & x_2^2 & x_3^2 \end{vmatrix} = \begin{vmatrix} x_1 & x_2 & x_3 \\ 1 & 1 & 1 \\ x_1^2 & x_2^2 & x_3^2 \end{vmatrix}$$

$$\begin{vmatrix} x_1 & x_2 & x_3 \\ 1 & 1 & 1 \\ x_1^2 & x_2^2 & x_3^2 \end{vmatrix} = \begin{vmatrix} x_1 & x_2 & x_3 \\ 1 & 1 & 1 \\ x_1^2 & x_2^2 & x_3^2 \end{vmatrix}$$

$$\frac{x^3}{e^{x^3}-1} \xrightarrow{x \to 0^+} 1$$

$$\frac{x^2}{x^3} \to +\infty$$

$$\frac{1}{\sqrt{x^2+1}+1} \xrightarrow{x \to 0^+} \frac{1}{2}$$

$$a^{1/3} - b^{1/3} = \frac{a-b}{a^{2/3} + b^{2/3} + (ab)^{1/3}}$$

D 5 D infinitesimo. Limitato

$\frac{1}{x^3}$ $\sin x$

$\sin x$ è $o(x^3)$

per $x \to +\infty$

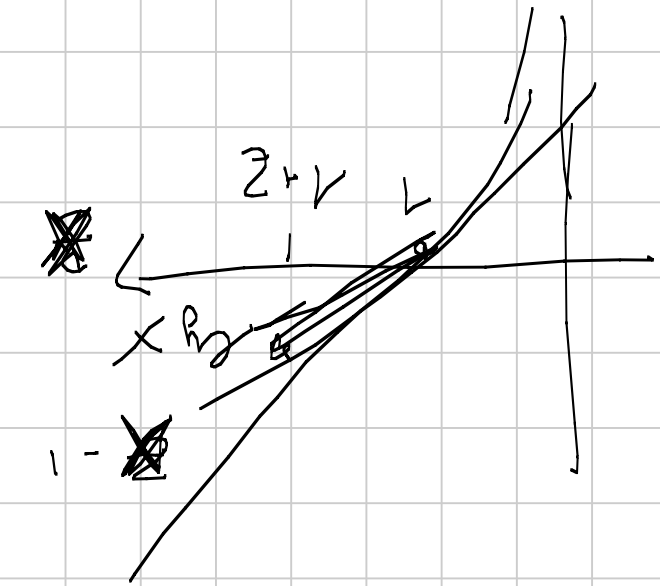
D 6

A) $f(x) = O(x^2)$ $x \to 0$

B) $f(x) = o(x)$ $x \to 0$

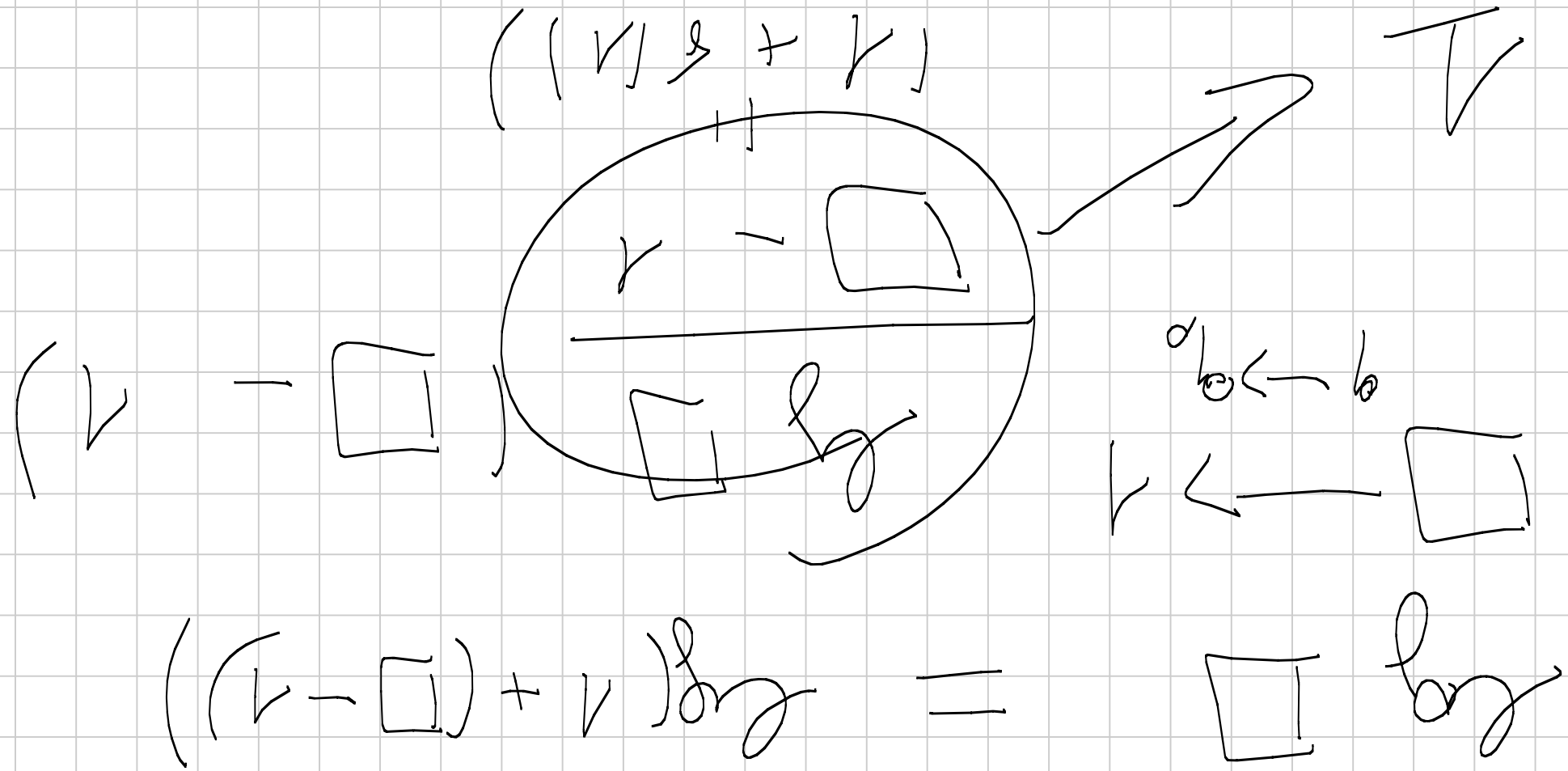
C) $f(x) = \delta(x)$ $x \to 0$

d) $f(x) = o(|x|^{3/2})$ $x \to 0$



$$\frac{z}{(z+1)\log}$$

$$\log\left(\frac{x_1 \cos\omega t + z}{x_2 \cos\omega t + 1}\right)$$



$$\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1} = \arctan(x+1) + C$$

$$\int \frac{dx}{x^2+2x+5} = \int \frac{dx}{(x+1)^2+4} = \frac{1}{2}\arctan\frac{x+1}{2} + C$$

$$\int \frac{dx}{x^2+2x+1} = \int \frac{dx}{(x+1)^2} = -\frac{1}{x+1} + C$$

$$\int \frac{dx}{x^2+2x} = \int \frac{dx}{(x+1)^2-1} = \frac{1}{2}\ln\left|\frac{x}{x+2}\right| + C$$

$$\frac{1 + \cos^2 x - 2 - \operatorname{arctg} \sin x}{2 + \operatorname{arctan} \sin x}$$

$$\frac{-(\sin x)^2 - \operatorname{arctg} \sin x}{2 + \operatorname{arctan} \sin x} = \frac{-\sin x + o(x)}{2 + o(1)}$$

$$f(x) = (1 + o(1)) \frac{1}{2 + o(1)} \cdot (-\sin x + o(x))$$

$$(1 + o(1)) \frac{1}{2 + o(1)} \left(- \frac{\sin x}{x} + \frac{o(x)}{x} \right) = \frac{-1 + o(1)}{x}$$

DS camino X

ESERCIZI A CASA

Ripassare al 6

Fare 7, 9, 10