Se $A_1 \dots A_{n-1}$ invertibili $\Rightarrow \exists!$ LU di A

dimostrazione per induzione sulla dimensione della matrice

$n=1$ $A = [a]$ $a = 1 \cdot x \Leftrightarrow x = a$

- Ipotesi induttiva
  Se $B \in \mathbb{R}^{(n-1)\times(n-1)}$ e $B_1 \dots B_{n-2}$ sono invertibili
  $\Rightarrow \exists!$ LU di B

$A \in \mathbb{R}^{n \times n}$ con $\underline{A_1 \dots A_{n-1} \text{ invertibili}}$

$$A = \begin{bmatrix} A_{n-1} & z \\ v^T & \alpha \end{bmatrix} = \begin{bmatrix} L_{n-1} & 0 \\ x^T & 1 \end{bmatrix} \begin{bmatrix} U_{n-1} & y \\ 0^T & \beta \end{bmatrix}.$$

$$\Leftrightarrow \begin{cases} A_{n-1} = L_{n-1} U_{n-1} \\ z = L_{n-1} y \\ v^T = x^T U_{n-1} \end{cases}$$

$$\alpha = x^T y + \beta$$

$$\det(A_{n-1}) = \det(L_{n-1})\det(U_{n-1}) = \det U_{n-1}$$

$$y = L_{n-1}^{-1} z$$

$$x^T = v^T U_{n-1}^{-1}$$

$$\alpha = x^T y + \beta \Rightarrow \beta = \alpha - x^T y$$

±		23

8 bit — 32 bit

$$2^0 + 2^1 + \dots + 2^7 = 2^8 - 1 = 255$$

$$p \in [-127, 128]$$

zero $[1\,0 \dots 0] \longrightarrow \underline{-127}$

rappr. esponente $\underline{p}$ esponente vero $\underline{p - 126}$

Condizionamento in norma $\underline{2}$

$$K(A) = \|A\|\,\|A^{-1}\|$$

$$\kappa_2(A) = \|A\|_2 \, \|A^{-1}\|_2 \qquad A \in \mathbb{R}^{n \times n}$$

$$\|A\|_2 = \sqrt{\rho(A^T A)}$$

<u>$A = A^T$</u> $\quad \|A\|_2 = \sqrt{\rho(AA)} = \sqrt{\rho(A^2)}$

$\lambda_1 \dots \lambda_n$ sono gli autovalori di A

$\lambda_1^2 \dots \lambda_n^2$ sono gli autovalori di A^2

$$|\lambda_n| = \max_{i=1\dots n} |\lambda_i| \qquad |\lambda_1| = \min_{i=1\dots n} |\lambda_i|$$

$$\|A\|_2 = \sqrt{\rho(A^2)} = \sqrt{|\lambda_n|^2} = |\lambda_n|$$

$$\|A^{-1}\|_2 = \sqrt{\rho\left(A^{-T} A^{-1}\right)} = \sqrt{\rho(A^{-1} A^{-1})}$$

$$A^{-T} = (A^{-1})^T = (A^T)^{-1} \qquad = \sqrt{\rho(A^{-2})}$$

$$=$$

$\frac{1}{\lambda_1^2} \dots \frac{1}{\lambda_n^2}$ sono gli autovalori di A^{-2}

$$\rho(A^{-2}) = \frac{1}{|\lambda_1|^2}$$

$$\|A^{-1}\|_2 = \sqrt{\frac{1}{|\lambda_1|^2}} = \frac{1}{|\lambda_1|}$$

$$K_2(A) = \|A\|_2 \|A^{-1}\|_2 = |\lambda_n| \cdot \frac{1}{|\lambda_1|}$$

$$= \left|\frac{\lambda_n}{\lambda_1}\right| = \frac{|\lambda_n|}{|\lambda_1|}$$

Per una matrice simmetrica il $K_2(A)$ è dato da modulo(λ_{max}) / modulo(λ_{min})

$$\begin{bmatrix} 0 & & & \\ 1 & \ddots & & \\ & \ddots & \ddots & \\ & & n & 0 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 0 & 1 & & \\ & \ddots & \ddots & \\ & & & n \\ 0 & & & \end{bmatrix} \begin{bmatrix} 0 & & & \\ 1 & \ddots & & \\ & \ddots & \ddots & \\ & & n & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & \cdots & \\ 0 & 1 & & \\ & & \ddots & \\ 0 & & 1 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix} \in \mathbb{R}^{4 \times 4}$$

$$\underline{Ax = 0} \qquad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 0 \Rightarrow x_2 + x_3 + x_4 = \\ x_1 + 2(x_2 + x_3 + x_4) = 0 \Leftrightarrow 2(\cancel{x_1 + \dots + x_4}) - x_1 = 0 \\ \cancel{x_1} + 2x_2 + 3(x_3 + x_4) = 0 \qquad 3(x_2 + x_3 + x_4) \underline{\underline{- x_2}} \\ x_1 + 2x_2 + 3x_3 + 4x_4 = 0 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \underline{\underline{U}}$$

$$\underline{\det U = 1}$$

$$A E_1 E_2 E_3 = U$$

$$A = U E_3^{-1} E_2^{-1} E_1^{-1}$$

$$E_3 E_2 E_1 A = U$$

$$A = \underbrace{\left(E_1^{-1} E_2^{-1} E_3^{-1}\right)}_{L} U$$

$$Ax = b$$

$$A E E^{-1} x = b$$

$$E A x = E b$$

$$\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

No

$$= \begin{bmatrix} 1 & \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \quad LU$$

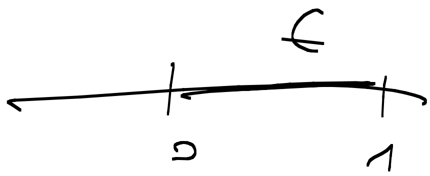
$$A = \begin{bmatrix} 1 & 1 \\ 1-\epsilon & 1 \end{bmatrix} \quad (\epsilon > 0)$$

$$K_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$$

$\epsilon = 0$ non è definit

$$\|A\|_\infty = \max\{2, |1-\epsilon|+1\}$$

$$= \underline{2}$$



$$\underline{\|A^{-1}\|_\infty}$$

$$\begin{bmatrix} 1 & 1 \\ 1-\epsilon & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1-\epsilon & 1 \end{bmatrix} \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1-\epsilon & 1 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{cases} a+c = 1 \\ (1-\epsilon)\, a + c = 0 \end{cases} \qquad \begin{cases} a+c = 1 \\ a - \epsilon a + c = 0 \end{cases}$$

$$\begin{cases} a+c = 1 \\ 1 = \epsilon a \iff a = \dfrac{1}{\epsilon} \end{cases} \qquad c = 1 - \frac{1}{\epsilon} = \frac{\epsilon - 1}{\epsilon}$$

$$\begin{bmatrix} 1 & 1 \\ 1-\epsilon & 1 \end{bmatrix} \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{cases} b + d = 0 \\ (1-\epsilon) b + d = 1 \Rightarrow -\epsilon b = 1 \end{cases}$$

$$b = -\frac{1}{\epsilon}$$

$$d = \frac{1}{\epsilon}$$

$$B = A^{-1} = \begin{bmatrix} \frac{1}{\epsilon} & -\frac{1}{\epsilon} \\ 1 - \frac{1}{\epsilon} & \frac{1}{\epsilon} \end{bmatrix}$$

$$K_\infty(B) = K_\infty(A^{-1})$$

$$= \max\left\{ \frac{2}{|\epsilon|}, \left|1 - \frac{1}{\epsilon}\right| + \left|\frac{1}{\epsilon}\right| \right\}$$

$$= \max\left\{ \frac{2}{|\epsilon|}, \frac{1}{\epsilon} - 1 + \left|\frac{1}{\epsilon}\right| \right\}$$

$$= \max\left\{ \frac{2}{\epsilon}, \frac{1}{\epsilon} - 1 + \frac{1}{\epsilon} \right\} = \frac{2}{\epsilon}$$

$$K_\infty(A) = 2 \cdot \frac{2}{\epsilon} = \frac{4}{\epsilon}$$

$(0 < \epsilon < 1)$

$$\lim_{\epsilon \to 0^+} K_\infty(A) = \lim_{\epsilon \to 0^+} \frac{4}{\epsilon} = \underline{+\infty}$$