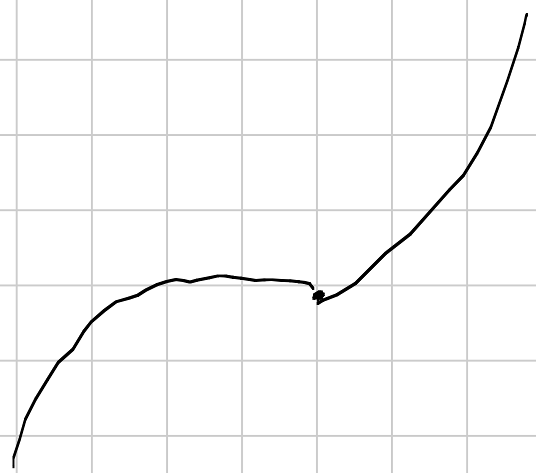
Es 15 nov 2013 _nzB seconda parte

Note Title 11/15/2019

STUDI DI FUNZIONE



P.to di flesso x_0 $\exists f'(x_0)$

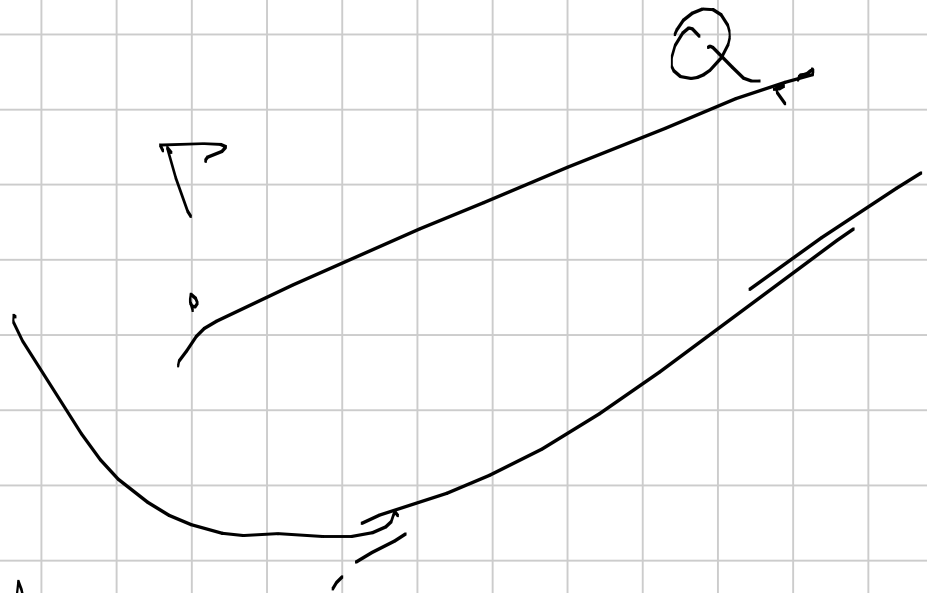
$x_0 - \delta < x < x_0$ f convessa [concava]

$x_0 < x < x_0 + \varepsilon$ f concava [convessa]

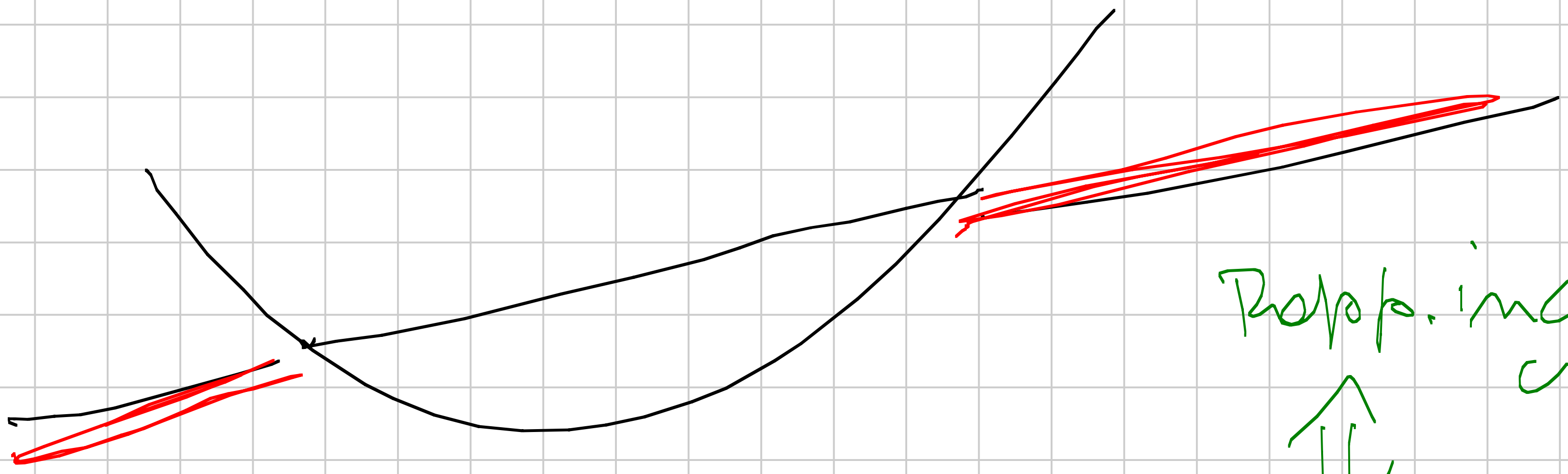
CONV.

$P(x, y)$: $f(x) \leq y$ allora il segmento

$Q(a, b)$ $f(a) \leq b$ tra P e Q sta nel sopragrafico



Le funz conv. hanno senso
su convessi



$$\frac{f(x)-f(x_0)}{x-x_0}$$

Rapp. increm. crescenti

$\Updownarrow$

f convesso, sì C convesso

$\Longleftrightarrow$ le corde con estremi nel grafico stanno sopra il grafico

$\Longleftrightarrow$ le co-corde con estr. nel graf stanno sotto di lui

Grafico di $\lg(x^2-1) = f(x)$

DOM $x^2 > 1$ $\quad (-\infty, -1) \cup (1; +\infty)$

PARI

SEGNO $f \geq 0 \iff x^2 - 1 \geq 1 \iff x^2 \geq 2$

$\iff x \geq \sqrt{2} \lor x \leq -\sqrt{2}$ $\quad$ N.B. $\pm\sqrt{2} \in$ dom

ASINTOTI vert. in 1^+ e -1^- unilateri

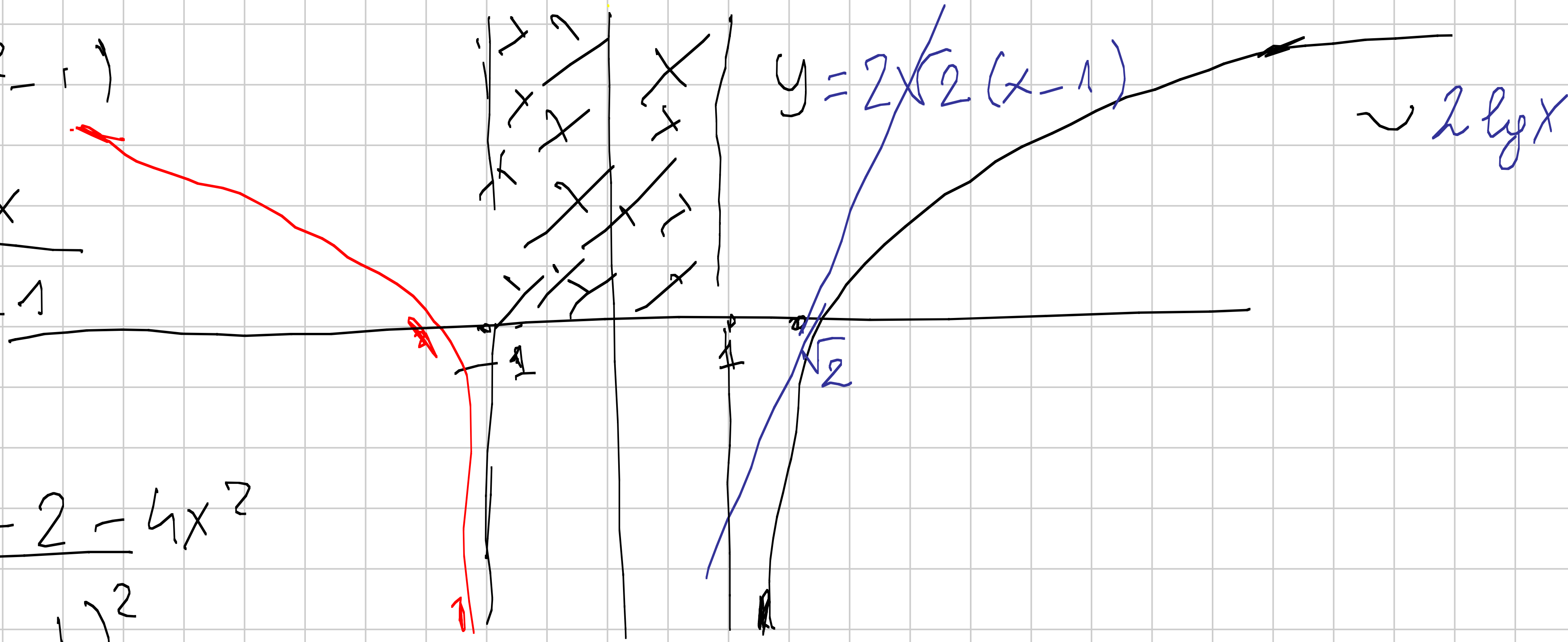
$f = \log(x^2-1)$

$f' = \dfrac{2x}{x^2-1}$

$f'' = \dfrac{2x^2-2-4x^2}{(x^2-1)^2}$

$\wedge$

0



PARI

Es1 settimo foglio

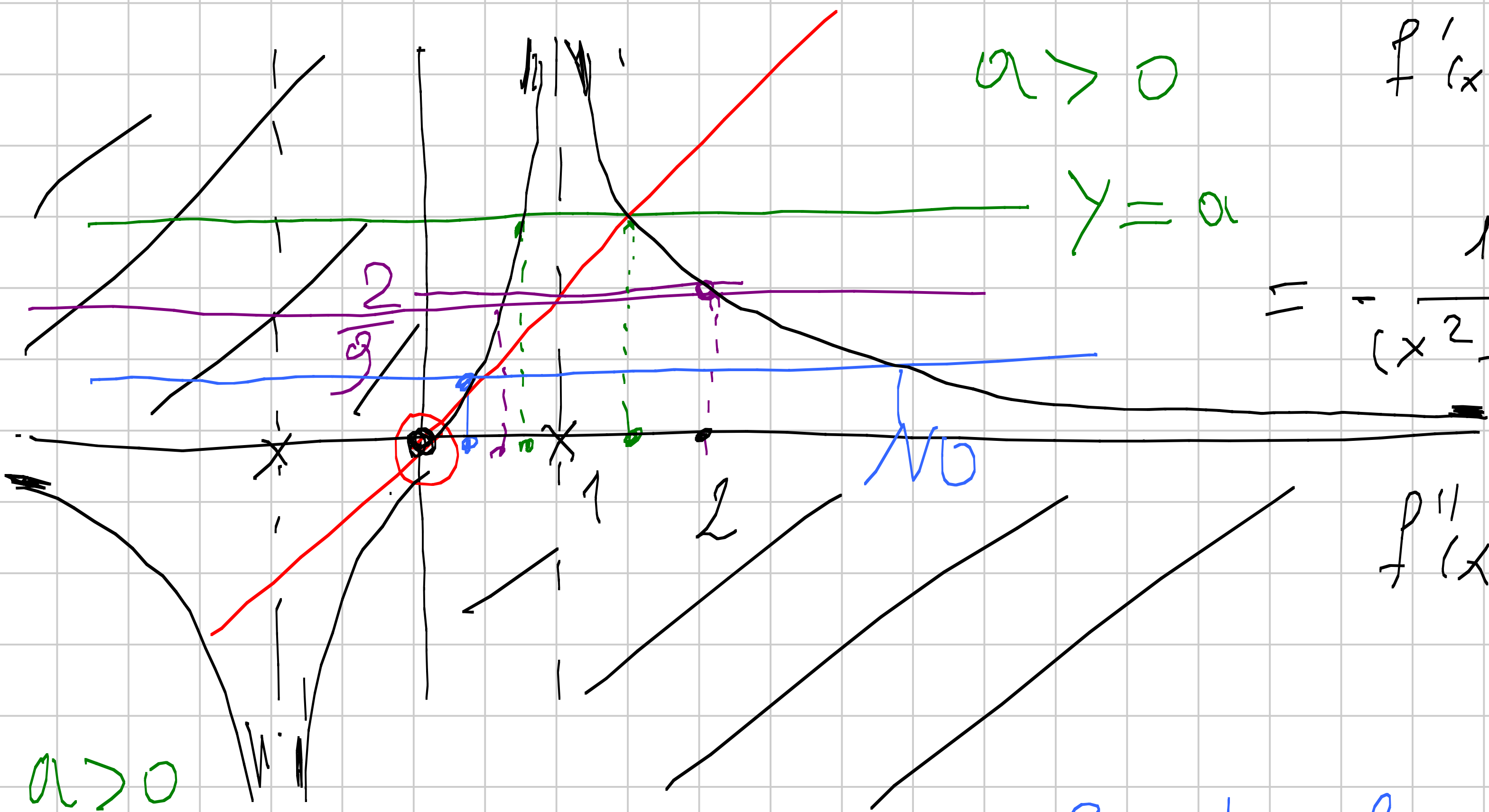
a) grafico di $f(x) = \frac{x}{(x^2-1)^2}$; dato $a>0$ quante sono le soluzioni di $\begin{cases} f(x) = a \\ x \leq 2 \end{cases}$?

DOMINIO: $x \neq 1$, $x \neq -1$

$(-\infty; -1) \cup (-1; 1) \cup (1; +\infty)$ $\quad x < -1$, $-1 < x < 1$, $x > 1$

SEGNO $f \geq 0 \iff x \geq 0$ $x \neq 1$

DISPARI $\quad$ ASINTOTI VERTICALI ± 1 ORIZ 0



$$f'(x) = \frac{(x^2-1)^2 - 4x^2(x^2-1)}{(x^2-1)^4} =$$

$$= -\frac{1+3x^2}{(x^2-1)^3} \qquad f'(0) = 1$$

$$f''(x) = \dots = 12x\,\frac{x^2+1}{(x^2-1)^4}$$

$$f(2) = \frac{2}{9}$$

$a > 0$

$$\begin{cases} f(x) = a \\ x \leq 2 \end{cases}$$

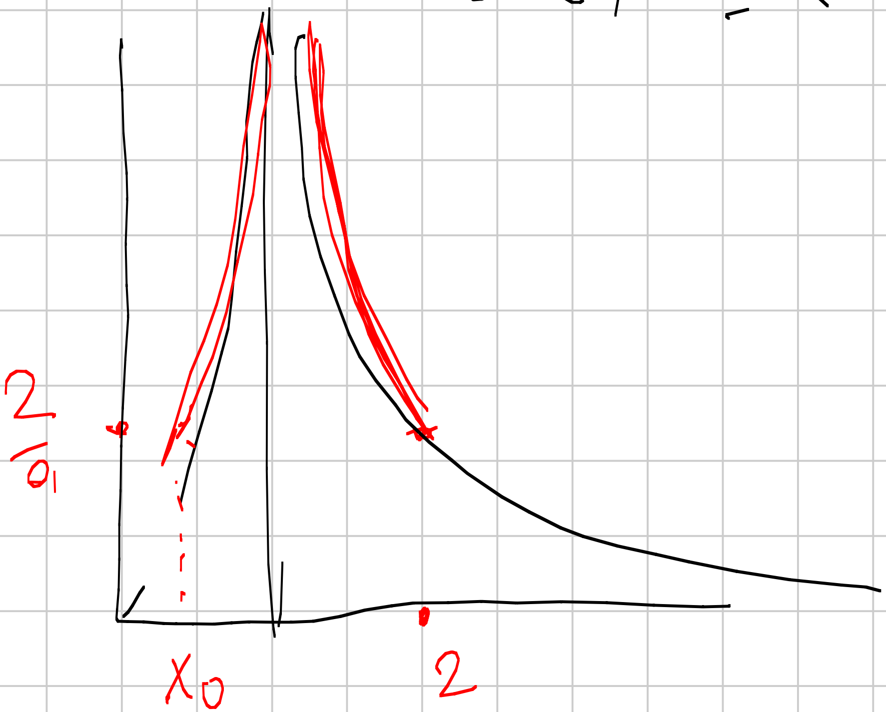
per $0 < a < \frac{2}{9}$ 1 sol.

per $a \geq \frac{2}{9}$ 2 sol.

b) $a > 0$ $\quad a > 2/9$

$m(a) = \min\{x : f(x) = a,\ x \le 2\}$

$M(a) = \max\{x : \text{- - - - -}\}$



$m(a) = g^{-1}(a)$

$g = f|_{(x_0;1)}$

$M(a) = h^{-1}(a)$

$h = f|_{(1,2]}$

$a \geq \frac{2}{9}$ $\quad M(a) = \max\{x : f(x) = a,\ x \leq 2\}$

$\boxed{M(\frac{2}{9}) = 2}$ $\quad = h^{-1}(a)$ $\quad h(x) = f(x) = \frac{x}{(x^2-1)^2}$

$h'(x) = -\frac{1+3x^2}{(x^2-1)^3}$ $\quad 1 < x \leq 2$

$h''(x) = 12x\frac{x^2+1}{(x^2-1)^4}$

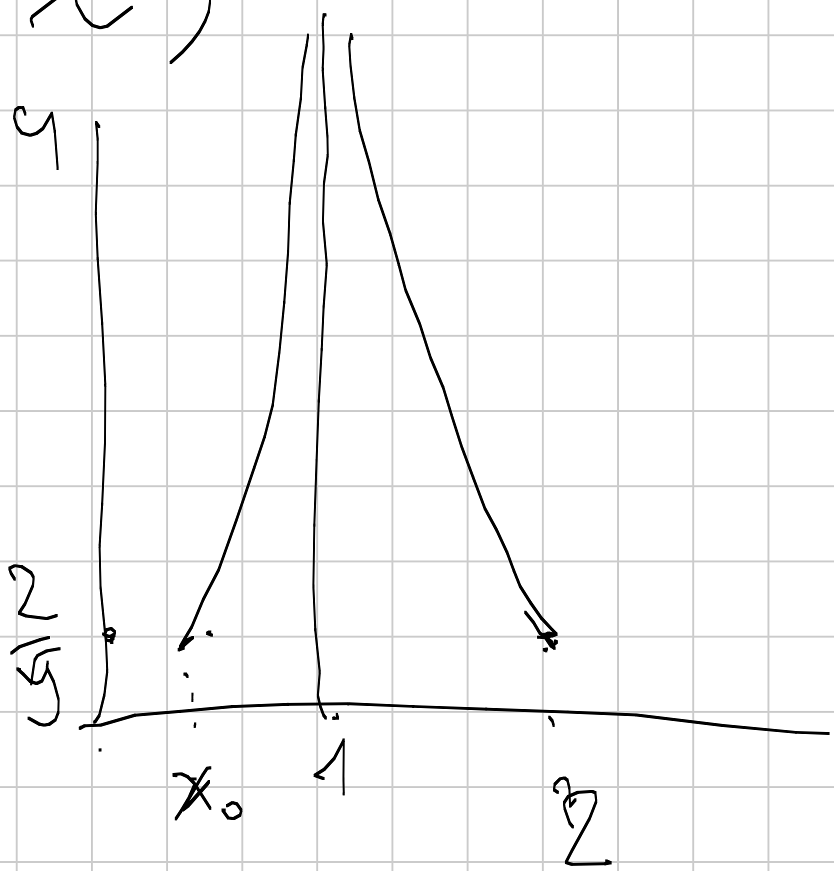
$M'(a) = \frac{1}{h'(M(a))}$ $\quad M'(\frac{2}{9}) = \frac{1}{h'(2)} = -\frac{27}{13}$

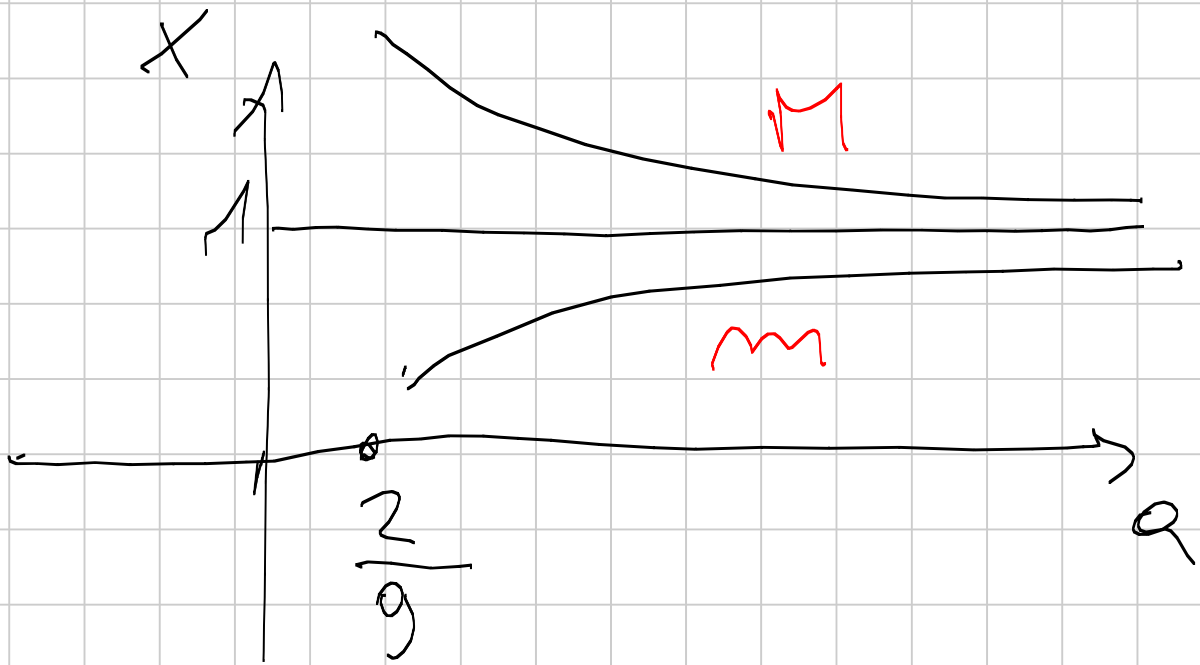
$M''(a) = -\frac{h''(M(a)) \cdot M'(a)}{[h'(M(a))]^2}$ $\quad M''(\frac{2}{9}) = -\frac{\frac{245}{27^2}\cdot\frac{27}{\cancel{13}}}{\frac{(27)^2}{(13)^2}}$

$$M(a) = 2 - \frac{27}{13}\left(a - \frac{2}{9}\right) + \frac{120 \cdot 27}{26}\left(a - \frac{2}{9}\right)^2$$

$$+ \left(a - \frac{2}{9}\right)^3$$

-C)



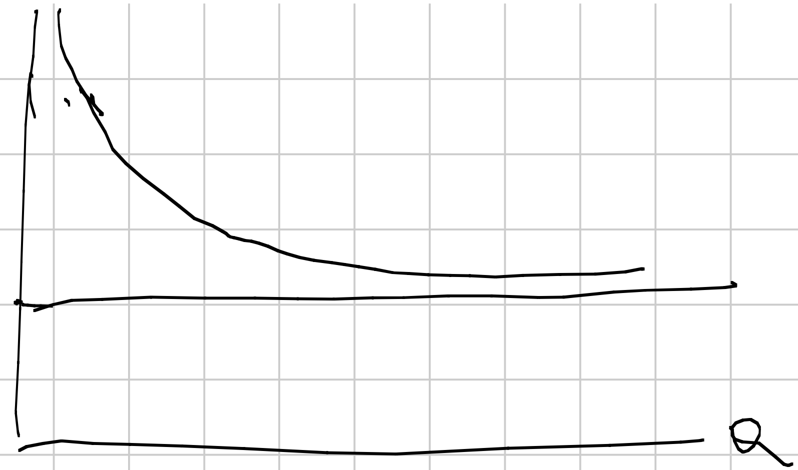


ma è crescente
l'altra decresce.

$m\left(\frac{2}{9}\right) = M\left(\frac{2}{9}\right) = 2$

$m'(2/9) > 0 \qquad M'\left(\frac{2}{9}\right) < 0$

d|,)



OSS. $\lim_{a\to 0^+} M(a) = +\infty \qquad M > 1$

$f(M(a)) = a \qquad \left(\frac{M(a)}{(M^2(a)-1)^2}\right) = a$

$\frac{1}{M^3} \frac{1}{\left(1-\frac{1}{M^2}\right)^2} = a$

$$\frac{1}{M^3} = a\left(1 - \frac{1}{M^2}\right)^2 = a + o(a)$$

$$M^3 = \frac{1}{a + o(a)}$$

$$\mathrm{pp}_{a \to 0}(M) = \frac{1}{\sqrt[3]{a}}$$