UNIVERSITÀ DI PISA

1993 IN SUPREMAE DIGNITATIS

UNIVERSITA' DEGLI STUDI DI PISA

DIPARTIMENTO DI MATEMATICA

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Primitive

## 1) ~~Integrali~~ notevoli

| | | | |
|---|---|---|---|
| | $e^x$ | | $e^x$ |
| $x>0$ | $x^a$ | $a \neq -1$ | $\frac{x^{a+1}}{1+a}$ |
| $x>0$ | $1/x$ | | $\log x$ |
| | $\sin x$ | | $-\cos x$ |
| | $\cos x$ | | $\sin x$ |
| | $\frac{1}{\cos^2 x}$ | | $\tan x$ |
| | $\frac{1}{\sqrt{1-x^2}}$ | | $\arcsin x$ / $-\arccos x$ |
| | $\frac{1}{\sqrt{1+x^2}}$ | | $\operatorname{arsinh} x$ / $\log(x+\sqrt{1+x^2})$ |
| | $\frac{1}{1+x^2}$ | | $\arctan x$ / $-\operatorname{arccot} x$ |

## 2) Teorema fondamentale:

$$f \in \mathcal{C}([a,b]),\ F(x) = \int_a^x f(t)\,dt \implies \boxed{\forall x \in [a,b]\ \exists F'(x) = f(x)}$$

$$\forall x \in [a,b]\ G'(x) = f(x) \implies \boxed{F - G \text{ costante su } [a,b]}$$

## 3) Parti:

$$g \in \mathcal{C}^1([a,b]),\ f \in \mathcal{C}([a,b]),\ F' = f \implies \boxed{\int_a^b f(t)g(t)\,dt = F(b)g(b) - F(a)g(a) - \int_a^b F(t)g'(t)\,dt}$$

e.g.

$$\int e^{\alpha x} \sin \beta x\, dx = e^{\alpha x}\frac{(\alpha \sin \beta x - \beta \cos \beta x)}{\alpha^2+\beta^2}$$ integrando due volte per parti

## 4) Sostituzione:

$$g \in \mathcal{C}^1([a,b]),\ f \in \mathcal{C}([a,b]) \implies \boxed{\int_a^b f(\varphi(t))\varphi'(t)\,dt = \int_{\varphi(a)}^{\varphi(b)} f(x)\,dx}$$

e.g. $\int_0^{\pi/2} \sin^m t \cos t\, dt = \frac{1}{m+1}$ ; $\int_0^{X} \sqrt{1-x^2}\, dx \overset{\substack{y \in [0,\pi] \\ x = \cos y}}{=} \int_{\arccos X}^{\pi} \sin^2 y\, dy \overset{\sin^2 y = \frac{1-\cos 2y}{2}}{=} \left[\frac{y - \sin y \cos y}{2}\right] =$

$$= \frac{\pi}{2} + \frac{\sqrt{1-X^2}\,X - \arccos X}{2}$$

## 5) Formule ricorsive.

$$\int \log^n x \, dx = x \log^n x - n \int \log^{n-1} x \, dx$$

$$\int x^n e^x dx = x^n e^x - \cancel{n \int x^{n-1} \log x \, dx} \quad n \int x^{n-1} e^x dx$$

$$\int x^n \sin x \, dx = -x^n \cos x + n \int x^{n-1} \cos x \, dx$$

$$\int x^\alpha \log^n x \, dx = \frac{x^{\alpha+1} \log^n x}{\alpha+1} - \frac{n}{\alpha+1} \int x^\alpha \log^{n-1} x \, dx \qquad \alpha \neq -1$$

## 6) Funzioni razionali elementari

$$\int \frac{x}{(x^2+1)^n} dx = \frac{1}{2} \int \frac{1}{y^n} dy = \begin{cases} \frac{1}{2} \log(1+x^2) & n = 1 \\ -\frac{1}{2(n-1)(x^2+1)^{n-1}} & n > 1 \end{cases}$$

$$\int \frac{dx}{x^n} dx = \begin{cases} \log|x| & n = 1 \\ -\frac{1}{(n-1)\, x^{n-1}} & n > 1 \end{cases}$$

$$\begin{cases} I_1 = \int \frac{dx}{1+x^2} = \operatorname{artan} x \\ I_n = \int \frac{dx}{(1+x^2)^n} = \frac{x}{2(n-1)(x^2+1)^{n-1}} + \frac{2n-3}{2(n-1)} \int \frac{dx}{(1+x^2)^{n-1}} = \frac{x}{2(n-1)(x^2+1)^{n-1}} + \frac{2n-3}{2(n-1)} I_{n-1} \quad n \end{cases}$$

( $\frac{1}{(1+x^2)^n} = \frac{1}{(x^2+1)^{n-1}} - \frac{x^2}{(x^2+1)^n}$ , il secondo addendo per parti.

## 7) Riduzione ad integrali razionali con sostituzione.

( $R(x_1, x_2)$ $R(x_1, x_2, x_3)$ siano funzioni razionali)

$\int R(\cos x, \sin x) dx$, $\tau = \tan \frac{x}{2}$ ; $\int R(\cosh x, \sinh x) dx$, $\tau = \tanh \frac{x}{2}$ ;

$\int R(x; \sqrt{1-x^2}) dx$, $\tau = \sqrt{\frac{1-x}{1+x}}$ ; $\int R(x \sqrt{x^2-1}) dx$, $z = \sqrt{\frac{x-1}{x+1}}$ ;

$\int R(x; \sqrt{1+x^2}) dx$, $\tau = x + \sqrt{x^2+1}$ o $\tau = \frac{\sqrt{x^2+1} - 1}{x}$ ; $\int R(x, \sqrt{ax^2+bx+c}) dx \ldots \uparrow$ ;

$\int R(x, \sqrt{ax+b}, \sqrt{\alpha x+\beta}) dx$, $z = \sqrt{\alpha x + \beta} \ldots \uparrow$ ;

$\int R\left(x, \sqrt[n]{\frac{ax+b}{\alpha x+\beta}}\right) dx$, $z = \sqrt[n]{\frac{ax+b}{\alpha x+\beta}} \ldots \uparrow$ .

## 8) ALCUNE FUNZIONI CON PRIMITIVA NON ELEMENTARMENTE ESPRIMIBILE

$n > 2$ $\frac{1}{\sqrt{a_0 + \cdots + a_n x^n}}$ , $\sqrt{a_0 + \cdots a_n x^n}$ ; $e^x/x$ ; $e^{x^2}$ ; $\frac{\sin x}{x}$

$\int_0^1 \frac{dz}{\sqrt{(1-z^2)(1-k^2 z^2)}}$ ; $\int \frac{dx}{\sqrt{\cos \alpha - \cos x}}$ , $\int \frac{dx}{\sqrt{1 - k^2 \sin^2 x}}$ .

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Calcolare:

$$\int_1^2 \frac{dx}{e^{2x}-e^x}, \quad \int_0^{\sqrt{\frac{3}{2}}} \frac{dx}{2x^2+3}, \quad \int_{\pi/4}^{\pi/2} \frac{\sin 3x + \cos x}{\sin x}\, dx, \quad \frac{1}{2}\int_0^2 \sin\sqrt{x}\, dx$$

$$\int_1^2 x\sqrt{e^x}\, dx, \quad \int_0^1 x \,\mathrm{artg}\, x, \quad \int_0^1 e^{\sqrt{x+1}}\, dx, \quad \int_0^{\pi} x e^{\frac{x^2}{\pi}}\, dx$$

$$\int_0^{\frac{\pi}{2}} x \cos\frac{x^2}{\pi}\, dx, \quad \int_{-1}^{1/2} x\sqrt{1-x^2}\, dx, \quad \int_{\pi/2}^{3/2\cdot\pi} \left\{ \frac{\sin^2 x}{x} + \sin 2x \log x \right\} dx$$

$$\int_0^1 \frac{\arccos x}{\sqrt{(1-x)2}}\, dx.$$

Trovare F:

$F'(x) = \mathrm{artg}\, x^2$ e $F(0)=0$; $F'(x) = \dfrac{\log(1+x^2)}{x^2}$ (TUTTE LE F)

$F'(x) = \dfrac{e^{2x}}{e^x+1}$ e $F(\log 2)=0$; $F'(x) = \dfrac{2x^2-1}{x^3-x}$ (TUTTE LE F)

$F'(x) = \dfrac{3x-3}{x^2-3x}$ (TUTTE LE F); $F'(x) = (x^2+2)e^x$ e $F(x) \geq 0$ (TUTTE LE F)

$F'(x) = \dfrac{x^3}{1+2x^2}$ (TUTTE LE F)