

Pseudoinverse

The solution of a least-squares problem $\min \|Ax - b\|$ is given by

$$x = (A^T A)^{-1} A^T b$$

(if A has full column rank).

Definition

The (Moore-Penrose) **pseudoinverse** of a matrix A with full column rank is $A^+ := (A^T A)^{-1} A^T$.

So we can write $x = A^+ b$ for the solution of a LS problem.

This Generalizes the concept of inverse — exists also for non-square A .

Not trivial: solution obtained by multiplying b by a certain matrix. (In particular, the solution of $\min \|Ax - (b_1 + b_2)\|$ is the sum of the two solutions of $\min \|Ax_1 - b_1\|$ and $\min \|Ax_2 - b_2\|$).

Note that $A^+ A = I_n$, but $AA^+ \neq I_m$ (there can be no matrix such that $AA^+ = I_m$, for rank reasons.)