$$\int \frac{dx}{1+x^2} = \operatorname{arctg} x + c.$$

$$\int \frac{dx}{k^2+x^2} = \qquad k \neq 0.$$

$$= \int \frac{dx}{k^2\left(1+\frac{x^2}{k^2}\right)} = \frac{1}{k^2}\int \frac{dx}{1+\frac{x^2}{k^2}}$$

$$\frac{x}{k} = t \qquad dx = k\,dt$$

$$= \frac{1}{k^{\cancel{2}}} \int \frac{\cancel{k}\,dt}{1+t^2} =$$

$$= \frac{1}{k} \operatorname{arctg} t + c =$$

$$= \frac{1}{k} \operatorname{arctg}\left(\frac{x}{k}\right) + c$$

3) nessuna radice reale.

$$x^2+px+q = x^2+px+\frac{p^2}{4}-\frac{p^2}{4}+q =$$

$$= \left(x+\frac{p}{2}\right)^2 + \left(q-\frac{p^2}{4}\right)$$

se non ci sono radici reali

$$q-\frac{p^2}{4} > 0.$$

quindi

$$\int \frac{dx}{x^2+px+q} = \int \frac{dx}{\left(x+\frac{p}{2}\right)^2+\left(q-\frac{p^2}{4}\right)} =$$

$$t = x+\frac{p}{2} \qquad dt = dx$$

$$= \int \frac{dt}{t^2+\left(q-\frac{p^2}{4}\right)} > 0 \quad \text{che è}$$

del tipo $\displaystyle\int \frac{dt}{t^2+k^2}$

$$\int \frac{dx}{x^2+2x+10} = \int \frac{dx}{x^2+2x+1-1+10} =$$

$$= \int \frac{dx}{(x+1)^2+9}$$ $\quad x+1=t \quad dx=dt$

$$= \int \frac{dt}{t^2+9} = \frac{1}{9}\int \frac{dt}{\frac{t^2}{9}+1}$$ $\quad s=\frac{t}{3} \quad t=3s$

$dt=3\,ds$

$$= \frac{1}{9}\int \frac{3\,ds}{s^2+1} = \frac{1}{3}\operatorname{arctg}(s)+c$$

$$= \frac{1}{3}\operatorname{arctg}\left(\frac{t}{3}\right)+c = \frac{1}{3}\operatorname{arctg}\left(\frac{x+1}{3}\right)+c$$

$$\int \frac{ax+b}{x^2+cx+d}\,dx = \frac{a}{2}\int \frac{2x+\frac{2b}{a}}{x^2+cx+d} =$$

$$= \frac{a}{2}\int \frac{2x+c}{x^2+cx+d} + \frac{\frac{2b}{a}-c}{x^2+cx+d}\,dx =$$

$$= \frac{a}{2}\log\left|x^2+cx+d\right| + \int \frac{b-\frac{c}{2}\cdot a}{x^2+cx+d}\,dx$$

Es: $$\int \frac{4x+5}{x^2+2x-1}\,dx = 2\int \frac{2x+\frac{5}{2}}{x^2+2x-1}\,dx$$

$$= 2\int \frac{2x+2-2+\frac{5}{2}}{x^2+2x-1}\,dx$$

$$= 2\int \frac{2x+2}{x^2+2x-1}\,dx + 2\int \frac{\frac{1}{2}}{x^2+2x-1}\,dx =$$

$$= 2\log\left|x^2+2x-1\right| + \int \frac{dx}{x^2+2x-1}$$