Note eser. 16/18 ott 2019

Note Title 10/16/2019

TEST 5

D. 1 $f(x) = e^{x^2 \cdot \lg(2+\cos x)}$

$d(e^y) = e^y \, dy$

$z = f(x) = e^y$

$y = g(x) = x^2 \cdot \lg(2+u)$

$u = h(x) = \cos x$

$\frac{dz}{dx} = \frac{dz}{dy} \frac{dy}{dx} = \frac{dz}{dy} \cdot \frac{d}{dx}(x^2 \cdot \lg(2+u))$

$$= \frac{\partial z}{\partial y}\left[\left(\frac{\partial}{\partial x} x^2\right) \lg(2+u) + x^2 \cdot \frac{\partial}{\partial x} \lg(2+u)\right]$$

$$= \frac{\partial z}{\partial y}\left(2x \lg(2+\cos x) + x^2 \frac{1}{2+u} \cdot \frac{\partial u}{\partial x}\right)$$

$$= e^y\left(2x \lg(2+\cos x) - \frac{x^2}{2+\cos x} \sin x\right)$$

$$= (2+\cos x)^2 \big($$

D2

Cele scanteie

$$\frac{f(x)-f(0)}{x-0} = \frac{\frac{x}{1-e^{1/x}} - 0}{x-0} =$$

$$= \frac{\frac{1}{1-e^{1/x}}}{x} \qquad y = \frac{1}{x} \qquad \frac{1}{1-e^{y}} \qquad y \to +\infty \quad y \to -\infty$$

$f'_+(0) = 0 \qquad f'_-(0) = -1$

$y \to 1 \quad \downarrow \quad y \to +\infty$

$1 \qquad 0$

$$\frac{1}{1 - e^{\frac{1}{x}}}$$

$x \to 0^+ \longrightarrow +\infty$

$\frac{1}{x} =: y \longrightarrow -\infty$

$x \to 0^-$

$\frac{1}{1 - e^y}$

$\xrightarrow[y \to +\infty]{} 0$

$\xrightarrow[y \to -\infty]{} \underline{1}$

D 3

$$y = f'(x_0)(x - x_0) + f(x_0) = r(x)$$

il grafico di $r(x)$ "è" la retta tangente nel punto $(x_0, f(x_0))$ al grafico di f

$$f'(1) = \left[\frac{1}{3} \left[\overline{\cos}(x-1) \right] \cdot (x-1)' \right]_{x=1}$$

$$\frac{dx}{dx} = 1 \quad 5$$

$$= \frac{1}{3}$$

$$y = \frac{1}{3}(x-1) + f(1)$$

$$0$$

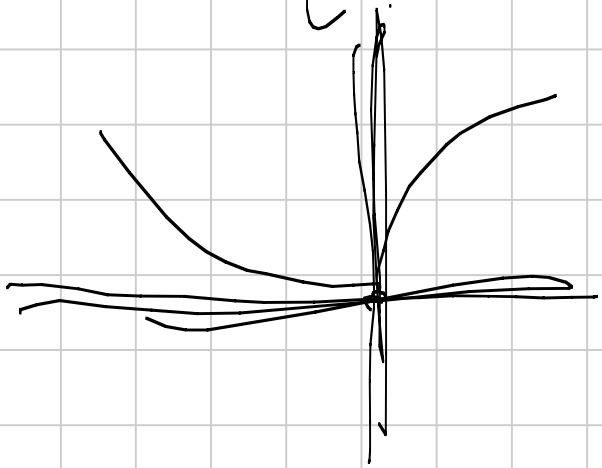
D. 4 LASCIATO PER CASA

Sugg. $f(x) = \sqrt[3]{\frac{e}{x} + \frac{e}{x^3} - \frac{e}{x^4}} =$

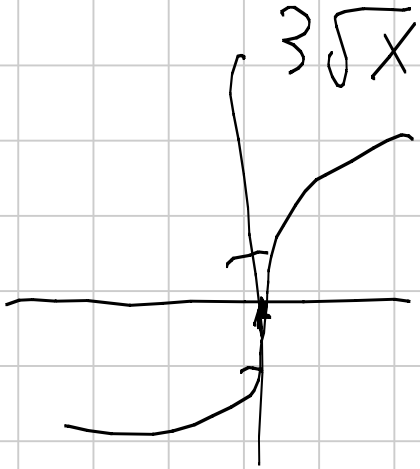
$\underset{y=\frac{1}{x}}{=} \left(e y + e y^3 - e y^4 \right)^{1/3}$

$$\frac{df}{dx}(x=1) = \frac{1}{3}\left(ey + ey^3 - ey^4\right)^{-2/3} \cdot \frac{d}{dy}\left(ey + ey^3 - ey^4\right) \cdot \frac{dy}{x}$$

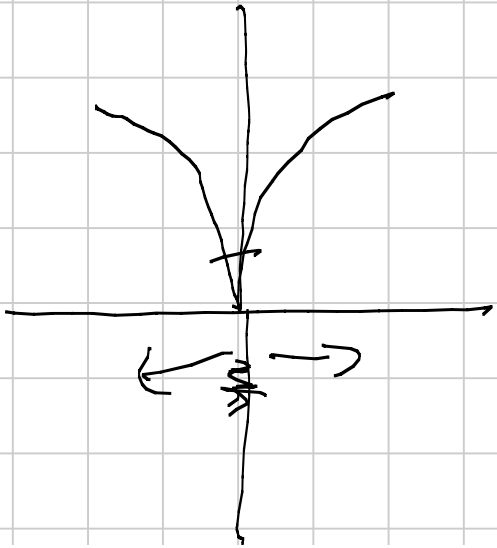
D5 $|x|^{3/2} \ln|x|$, 0



$\sqrt[3]{x}$



$\sqrt{|x|}$



$$|x|^{1/4} \ln|x| \xrightarrow[x \to 0]{} 0$$

$$f(x) = o\left(|x|^{5/4}\right)$$

$$= |x| \cdot o\left(|x|^{1/4}\right)$$

$$\frac{f(x)}{x} = \frac{x}{|x|} o\left(|x|^{1/4}\right) \longrightarrow 0$$

$$\frac{3}{2} - l = \frac{1}{4}$$

$$\frac{6}{4} - \frac{1}{4} = l$$

$$\frac{5}{4} = l$$

$f(x)$ è $o(x)$ $x \to 0$

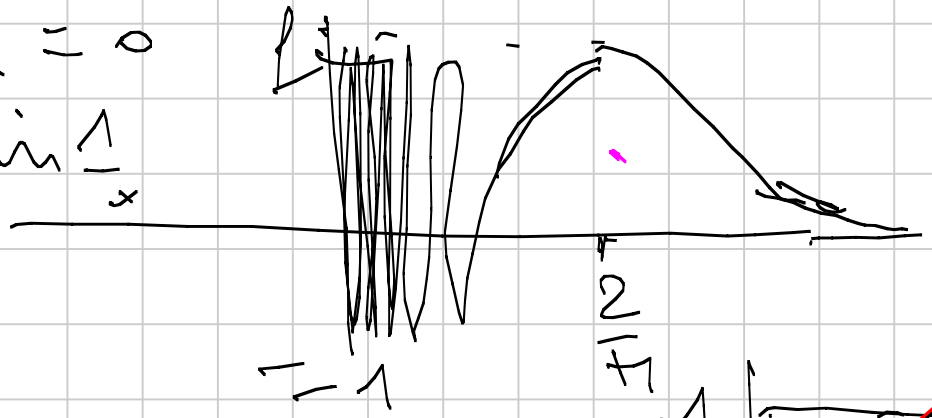
cioè $\dfrac{f(x)}{x} \xrightarrow[x \to 0]{} 0$

cioè $\exists f'(0) = 0$

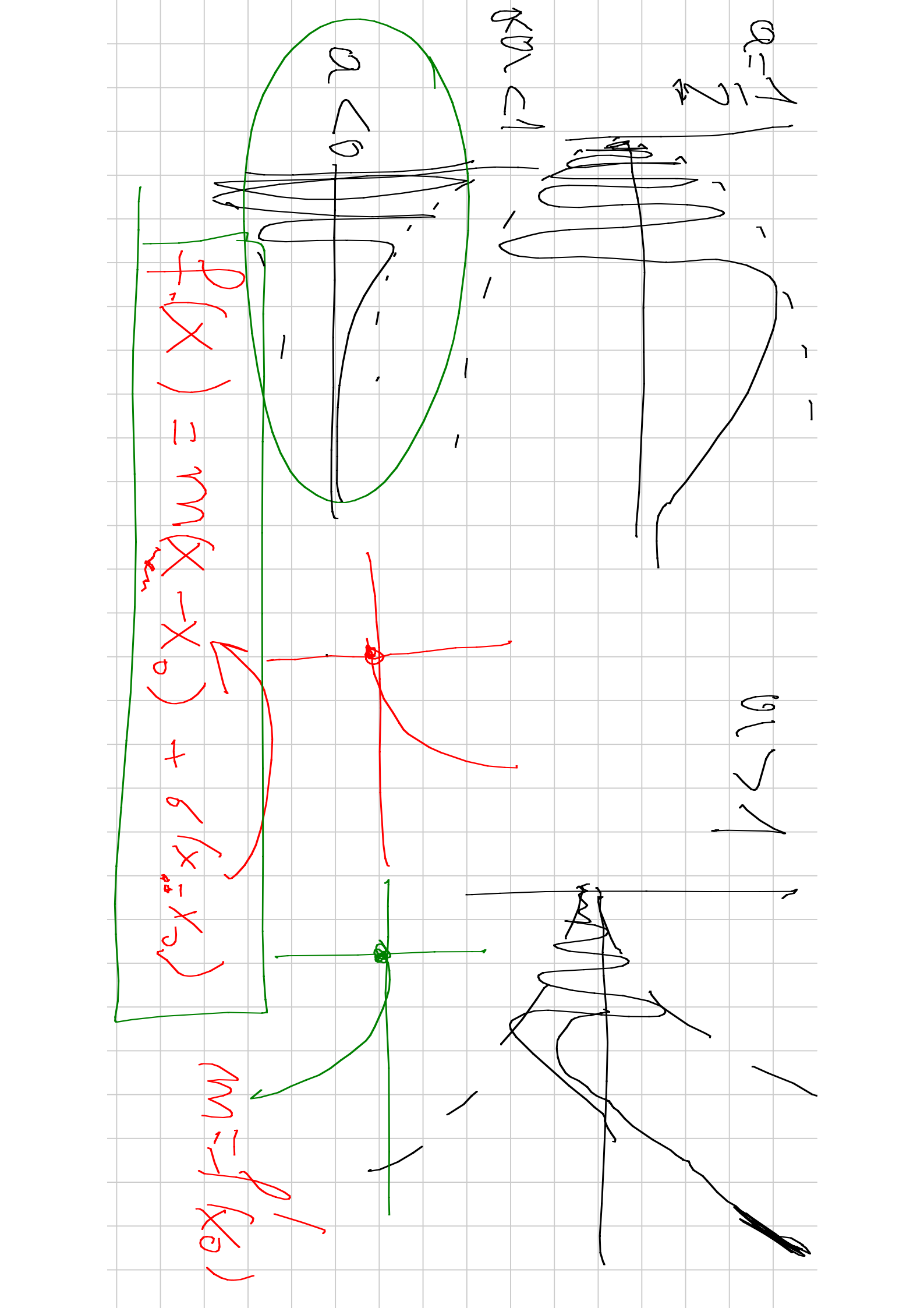
D6 $f(x) = \begin{cases} |x|^a \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

$a = 0$

$\sin\frac{1}{x}$



$a = 1 \quad |x| \sin\frac{1}{x}$



$$f(x) = \begin{cases} |x|^\alpha \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$x > 0$

$$\frac{d}{dx}\left(x^\alpha \sin\frac{1}{x}\right) = \alpha x^{\alpha-1}\sin\frac{1}{x} + x^\alpha \left(\cos\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)$$

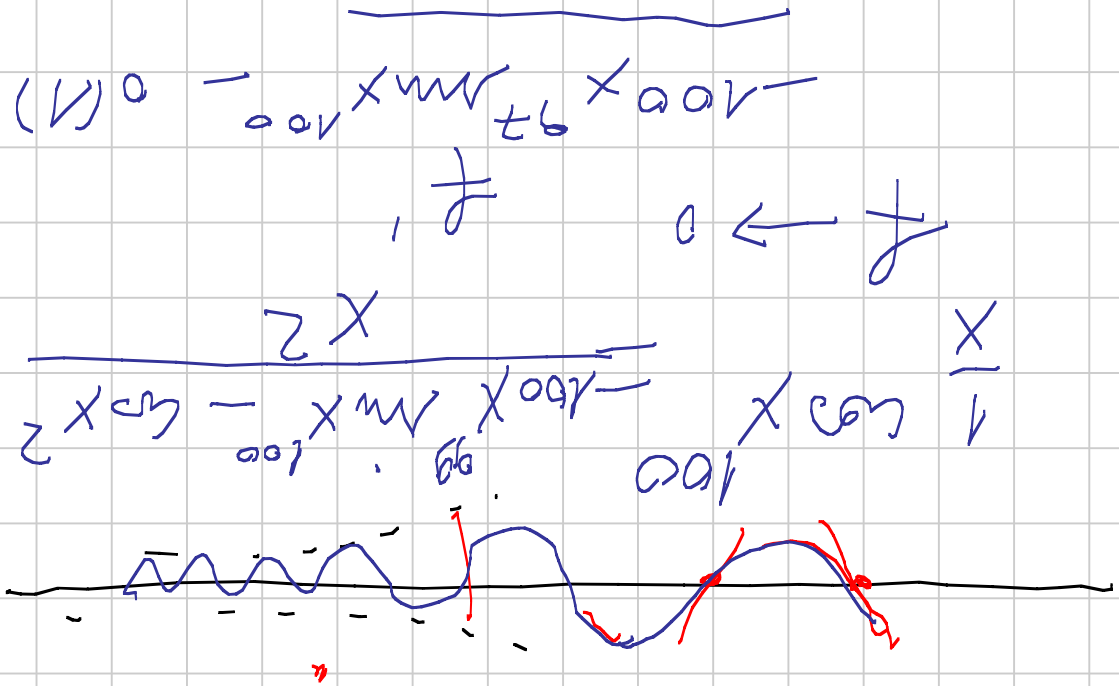
$$= \alpha x^{\alpha-1}\sin\frac{1}{x} - \boxed{x^{\alpha-2}\cos\frac{1}{x}}$$

$g(x)$

NOTA per $\alpha < 2$ (anche 2)

$g\left(\frac{1}{2k\pi}\right) = +(2\pi)^{\alpha-2} k^{\alpha-2} \longrightarrow +\infty$

$g\left(\frac{1}{2k\pi+\pi}\right) = -(\pi)^{\alpha-2}(2k+1)^{\alpha-2} \longrightarrow -\infty$



D8 a $\cos 0$

D9 $x^2 \sqrt{x^2-2x+1} = 6(x)$

$x \to 0$

$\frac{f(x) - f(x_0)}{x - x_0} = \frac{f(x)}{x - x_0} \to$

$x^2|x-1|$

$\begin{cases} x^2(x-1) & x>1 \\ x^2(1-x) & x\leq 1 \end{cases}$