Note Title — 22 NOVEMBRE — 11/22/2019

IX D.1

$$a_n = e^{4n \log \sin \sqrt[4]{|\sin n|}}$$

? $\sin \sqrt[4]{|\sin x_n|} \longrightarrow 1$? NO

$|\sin n| \leq 1$

$\sin \sqrt[4]{|\sin n|} \leq \sin 1 < 1$

$\neq 1$

D2 D3 D4 OK

D5 criterio rapporto

$$\frac{(3n+3)!}{(n+1)!\,(2n+2)!} \cdot \frac{n!\,(2n)!}{(3n)!} =$$

$$= \frac{(3n+3)(3n+2)(3n+1)}{(n+1)(2n+2)(2n+1)} = \frac{9n^3+\dots}{4n^3+\dots} \to 9/4$$

ES. 1 $\quad a_n = \dfrac{\sum_{k=1}^{n} \lg k}{n^2} =$

$$0 \le \frac{\lg 1 + \dots + \lg n}{n^2} \le \frac{n \lg n}{n^2} = \frac{\lg n}{n} \to 0$$

DOMANDA

$$\frac{\lg(n!)}{n} = \frac{\lg 1 + \dots + \lg n}{n} \longrightarrow +\infty$$

$$\frac{\lg(n+1)!}{n+1} \cdot \frac{n}{\lg n!} = \frac{n}{n+1} \qquad \frac{\boxed{\sum^{n+1} \lg k}}{\lg n!} = \qquad \lg n! + \lg n+1$$

NOTA $a_n > 0$

$$\frac{a_{n+1}}{a_n} \longrightarrow l \implies \sqrt[n]{a_n} \longrightarrow l$$

$$\sqrt[n]{a_n} = \sqrt[n]{a_0}\sqrt[n]{\frac{a_1}{a_0} \cdot \ldots \cdot \frac{a_{n-2}}{a_{n-3}} \frac{a_{n-1}}{a_{n-2}} \frac{a_n}{a_{n-1}}}$$

1

$$\alpha_n \longrightarrow \ell \Longrightarrow \frac{\alpha_1+\dots+\alpha_n}{n} \longrightarrow \ell$$

$$\varepsilon > 0 \quad \exists\, n_0 \quad \forall n > n_0 \quad \ell-\varepsilon \le \alpha_n \le \ell+\varepsilon$$

$$\left(\alpha_n \geq \frac{1}{\varepsilon} \right)$$

$$\left| \frac{\alpha_1+\dots\alpha_n}{n} - \ell \right| = \left| \frac{(\alpha_1-\ell)+\dots(\alpha_n-\ell)}{n} \right| \le$$

$$\le \left| \frac{(\alpha_1-\ell)+\dots+(\alpha_{n_0}-\ell)}{n} \right| + \frac{n-n_0}{n}\cdot\varepsilon \le \varepsilon$$

ES 2

$$\frac{n^{2n}}{(2n)!} = a_n$$

CRITERIO RADICE r . . .

$$\frac{n^2}{\sqrt[n]{(2n)!}} \leq \frac{n^2}{\sqrt[n]{n!}} \qquad \sqrt[n]{\frac{n^{2n}}{n!}}$$

$$\frac{n^{2n}}{(2n)!} = \frac{n}{2n} \cdot \frac{n}{n+1} \cdot \frac{n}{n-1} \cdot \ldots \cdot \frac{n}{1}$$

$$\frac{(n+1)^{2(n+1)}}{(2n+2)!} \cdot \frac{(2n)!}{n^{2n}} = \frac{\left(\frac{n+1}{n}\right)^{2n} (n+1)^2}{(2n+2)(2n+1)}$$

e^2

$\frac{1}{4}$

$$\longrightarrow \frac{e^2}{4} > 1$$