Esercitazione 22/07

$$f(x) = e^{x^2-1} + x - 1 = 0$$

$$f \in C^\infty(\mathbb{R})$$

$$\lim_{x \to +\infty} e^{x^2-1} + x - 1 = +\infty$$

$$\lim_{x \to -\infty} \underbrace{e^{x^2-1}}_{\to +\infty} + \underbrace{x}_{\to -\infty} - 1 = \lim_{x \to -\infty} e^{x^2-1} \cdot \left(1 + \underbrace{\frac{x - 1}{e^{x^2-1}}}_{\to 0}\right)$$

$$= +\infty$$

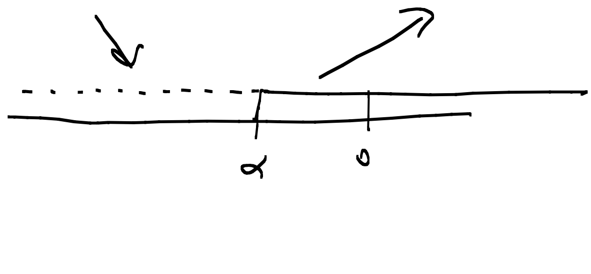
$$f'(x) = e^{x^2-1} \cdot 2x + 1$$

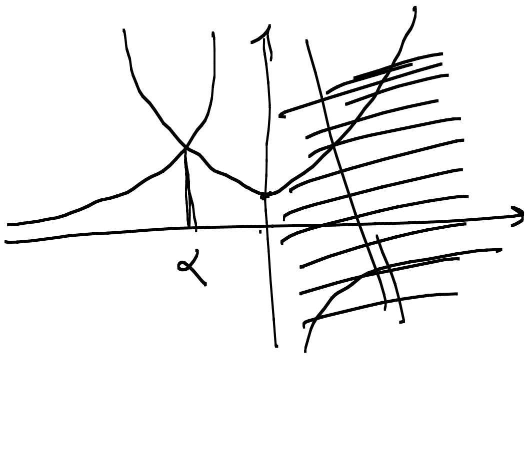
$$f'(x) > 0 \quad \forall x \geq 0$$

$x < 0$ $\quad e^{x^2-1} \cdot 2x + 1 \geq 0 \iff$

$$e^{x^2-1}\, 2x \geq -1 \iff$$

$$e^{x^2-1} \leq -\frac{1}{2x}$$





$$f''(x) = e^{x^2-1}\cdot 4x^2 + e^{x^2-1}\cdot 2$$

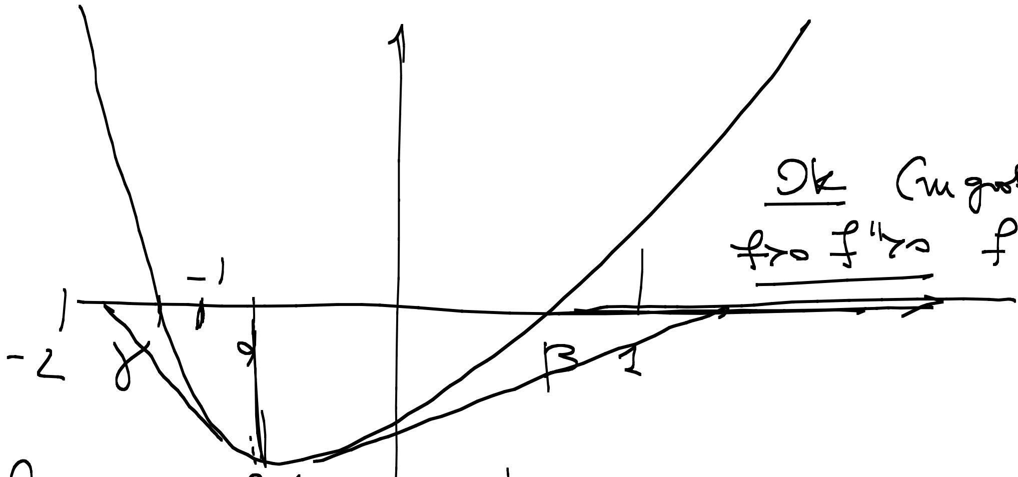
$$= e^{x^2-1}\cdot(4x^2+2) > 0 \quad \forall x \in R$$

Ok

$f>0$

$f''>0$

$f'<0$



Ok (in grande)

$f>0 \quad f''>0 \quad f'<0$

$f(\alpha)$? $\qquad f(0) = e^{-1}+0-1 = \frac{1}{e}-1 < 0$

$\beta \in [0, 1] \qquad f(1) = e^0 + 1 - 1 = 1$

$\gamma \in [-2, 0] \qquad f(-1) = e^0 - 1 - 1 = -1$

$f(-2) = e^{4-1} - 2 - 1 = e^3 - 3 > 0$

$x_0 \geq \beta \qquad x_k \to \beta$ (Teorema di convergenza in grande)

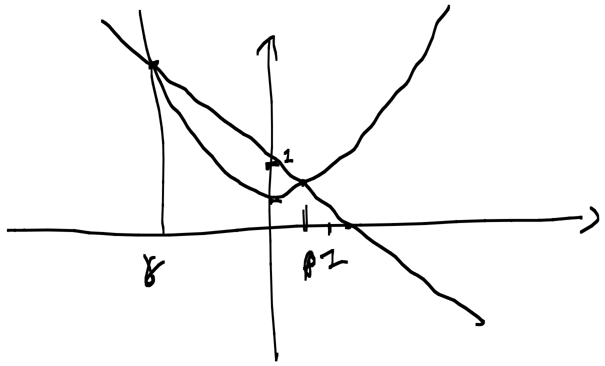
$\alpha < x_0 \leq \beta \Rightarrow x_1 \geq \beta \qquad x_k \to \beta$

$x_0 \leq \gamma \qquad x_k \to \gamma$ (Teorema di convergenza in grande)

$\gamma \leq x_0 < \alpha \Rightarrow x_1 \leq \gamma \Rightarrow$ Guangen 8

$$f(x) = e^{x^2-1} + x - 1 = 0$$

$$e^{x^2-1} + x - 1 = 0 \iff e^{x^2-1} = 1 - x$$



$$B = \begin{bmatrix} 1 & & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & & & 1 \\ \vdots & \ddots & & \vdots \\ \vdots & & 1 & 1 \\ 1 & \cdots & 1 & 2 \end{bmatrix}$$

B invertible $\rightsquigarrow$ $\det B \neq 0$

$$B = \begin{bmatrix} 1 & & & 1 \\ \vdots & \ddots & & \vdots \\ 1 & \cdots & 1 & 1 \\ 1 & \cdots & 1 & 2 \end{bmatrix} = \left[\begin{array}{ccc|c} 1 & & & \\ \vdots & \ddots & & 0 \\ 1 & \cdots & 1 & \\ \hline 1 & \cdots & 1 & 1 \end{array}\right] \left[\begin{array}{c|c} I_{n-1} & \begin{matrix} 1 \\ 0 \\ \vdots \\ 0 \end{matrix} \\ \hline & 1 \end{array}\right] -$$

$$\begin{bmatrix} 1 & & \\ \vdots & \ddots & \\ 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ v_{n-1} \end{bmatrix} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

$v_1 = 1$

$v_2 + v_1 = 1 \Rightarrow v_2 = 0$

$v_3 + v_2 + v_1 = 1 \Rightarrow v_3 = 0$

$$1 + v_n = 2 \qquad v_n = 1$$

$\det B = \det U = 1$ ok per invertibilità

$$B = A + \begin{bmatrix} 0 & \\ & 1 \end{bmatrix} \qquad M = \begin{bmatrix} 1 & & \\ \vdots & \ddots & \\ 1 & \cdots & 1 \end{bmatrix} \quad N = \begin{bmatrix} 0 & \begin{matrix} -1 \\ \vdots \\ -1 \end{matrix} \end{bmatrix}$$

$M - N = B$

G-S $\quad M = \begin{bmatrix} 1 & & & \\ \vdots & \ddots & & \\ & & 1 & \\ 1 & \cdots & 1 & 2 \end{bmatrix}$

$$P = M^{-1} N = \begin{bmatrix} 1 & & \\ \vdots & \ddots & \\ 1 & \cdots & 1 \end{bmatrix}^{-1} \left[0 \,\middle|\, \cdots \,\middle|\, 0 \,\middle|\, \begin{matrix} -1 \\ \vdots \\ -1 \end{matrix} \right]$$

$$= \left[0 \,\middle|\, \cdots \,\middle|\, 0 \,\middle|\, M^{-1} \begin{bmatrix} -1 \\ \vdots \\ -1 \end{bmatrix} \right]$$

$$M^{-1} \begin{bmatrix} -1 \\ \vdots \\ -1 \end{bmatrix} = x \iff Mx = \begin{bmatrix} -1 \\ \vdots \\ -1 \end{bmatrix} \qquad x = \begin{bmatrix} -1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & \cdots & 0 & \begin{matrix} -1 \\ 0 \\ \vdots \\ 0 \end{matrix} \end{bmatrix}$$

$$\|P\|_1 = 1 = \|P\|_\infty \qquad \|P\|_2 = \,?$$

P triangolare superiore $\Rightarrow$ i suoi autovalori stanno sulla diagonale principale $\Rightarrow$ $\lambda_1 = \dots = \lambda_n = 0 \qquad \rho(P) = 0$

Il metodo è convergente. ||

$$P = \begin{bmatrix} & -1 \\ 0 & \end{bmatrix} \qquad P^2 = \begin{bmatrix} 0 & -1 \\ & \end{bmatrix} \begin{bmatrix} 0 & -1 \\ & \end{bmatrix} = 0$$

$$e_1 = P \cdot e_0 \qquad e_2 = P e_1 = P^2 e_0 = 0 \quad (x_2 = x)$$

$$\rho(P) = 0 \;\not\Rightarrow P = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad P = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\exists\, k$ t.c. $P^k = 0$

$$e_{k+1} = P e_k = P^2 e_{k-1} = \dots = P^{k+1} e_0$$

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$A_{11} \in \mathbb{R}^{k \times k}$ $A_{12} \in \mathbb{R}^{k \times n-k}$

$A_{21} \in \mathbb{R}^{(n-k) \times k}$ $A_{22}^{(n-k) \times (n-k)}$

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$A_{11} \in \mathbb{R}^{k \times k}$

$A_{22} \in \mathbb{R}^{(n-k) \times (n-k)}$

$$A \cdot B = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

$$= \begin{bmatrix} (A_{11} B_{11} + \underline{A_{12} B_{21}} \\ \end{bmatrix}$$

$$A = \begin{bmatrix} A_{11} & 0 \\ A_{12} & B_{22} \end{bmatrix}$$

$$\underline{\det A = \det A_{11} \cdot \det A_{22}}$$

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$A = \left[\begin{array}{c|c} \hat{A} & v \\ \hline w^T & \alpha \end{array}\right] = \left[\begin{array}{c|c} L & 0 \\ \hline x^T & 1 \end{array}\right] \left[\begin{array}{c|c} U & z \\ \hline 0 & \beta \end{array}\right]$$

$\hat{A} = L \cdot U \quad \sim \quad = L2 + 0 \cdot \beta \doteq L2 \ldots$

$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \qquad \det A = \cancel{\det A_{11} \det A_{22}} - \det A_{12} \det A_{21}$

$A = \begin{bmatrix} A_{11} & 0 \\ A_{12} & A_{22} \end{bmatrix} \qquad \underline{\det(A - \delta I)} = \det \begin{pmatrix} A_{11} - \delta I & 0 \\ A_{12} & A_{22} - \delta I \end{pmatrix}$

$\det(\underline{A_{11} - \delta I}) \cdot \det(\underline{A_{22} - \delta I})$

$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad \det(A - \delta I) \rightsquigarrow \underline{NO}$

$f(x) = (x-1)e^{x+1} - a = 0 \qquad (a > 0)$

$f \in C^\infty(\mathbb{R})$

$\lim_{x \to +\infty} f(x) = +\infty$

$\lim_{x \to -\infty} (x-1)e^{x+1} - a = -a$

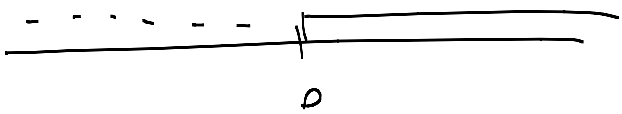
$\lim_{x \to -\infty} (x-1)e^{x+1} = \lim_{x \to -\infty} \frac{x-1}{e^{-x-1}} = !$

$$= \lim_{x \to -\infty} \frac{1}{-e^{-x-1}} = 0$$

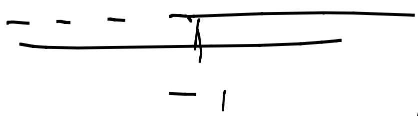
$$f(x) = (x-1)e^{x+1} - a$$

$$f'(x) = e^{x+1} + (x-1)e^{x+1}$$

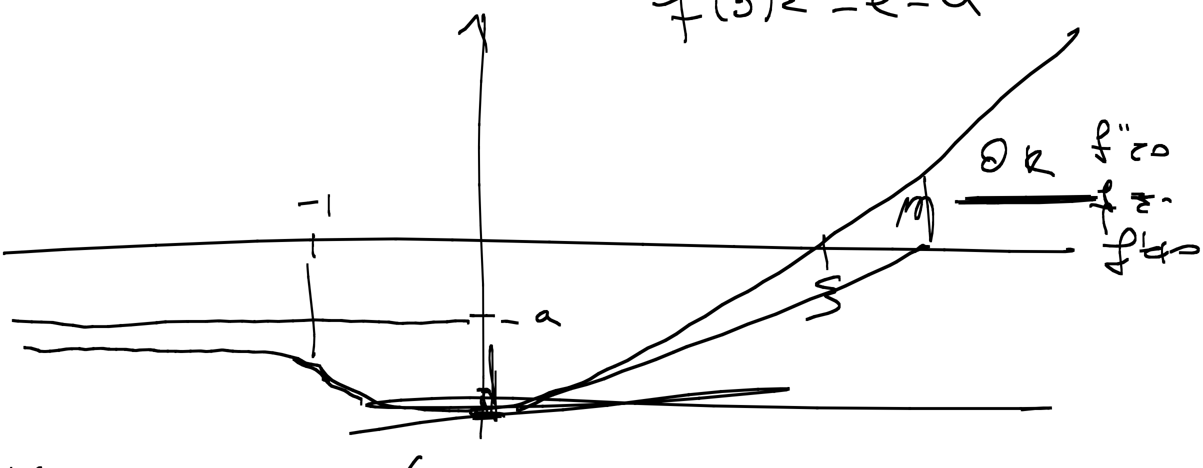
$$= e^{x+1} \cdot (1 + x - 1) = x e^{x+1}$$



$$f''(x) = e^{x+1} + x \cdot e^{x+1} = e^{x+1} \cdot (1+x)$$



$$f(0) = -e - a$$



$$\forall x_0 > 0 \qquad x_k \to \xi$$