NOTE ES: 13-15 nov. 2019

Note Title 11/13/2019

VII Foglio

D. 1 $x_0 = 1$ $n = 2$ $f(x) = e^x$

SVILUPPO

B) NO POICHÉ IL RESTO È $o(x^2)$ $x \to 1$

NOTA $x - 1 \neq o(x-1)$ per $x \to 1$

ma $x - 1 = o(x^2)$ per $x \to 0$

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + f''(x_0)/2\,(x - x_0)^2 + o((x - x_0)^2)$$

$$e^x = e + \underline{e(x-1)} + \frac{e}{2}(x-1)^2 + o((x-1)^2)$$

$$e^x = \cancel{e} - \cancel{e} + \cancel{ex} + \frac{e}{2}x^2 - \cancel{e}x + \frac{e}{2} + o(x-1)^2$$

$$e^x = \frac{e}{2} + \frac{e}{2}x^2 + o(x-1)^2$$

D. 2 f ha p.p. in 0 se $\exists \alpha, \beta \cdot \frac{f(x)}{\alpha x^\beta} \xrightarrow[x \to 0]{} 1$

$$\log(1-5x^2) + 5[\log(1-x)]^2$$

$x \to 0$

$\log(1+t)$ $t \to 0$

$$\log(1+t) \underset{t \to 0}{=} t - \frac{t^2}{2} + \frac{t^3}{3} + o(t^4)$$

$t = -5x^2$

$t = -x$

$t = -5x^2$

$t = -x$

$$-5x^2 - \frac{25x^4}{2} + O(x^6) \; \dots\dots$$

$$+5\left[-x - \frac{x^2}{2} + O(x^3)\right]^2$$

$$\cancel{-5x^2} - \frac{25}{2}x^4 + O(x^6) + \cancel{5x^2} + \frac{5}{4}x^4 + O(x^3)$$

NON VA

$$\cancel{-5x^2} - \frac{25}{2}x^4 - \frac{5^3}{3}x^6 + O(x^8) + 5\left[-x - \frac{x^2}{2} - \frac{x^3}{3} + O(x^4)\right]^2$$

$$5x^3 + O(x^4)$$

$$\cancel{5x^2} + \frac{5}{4}x^4 + \frac{5}{9}x^6 + 5x^3 + O(x^4)$$

D.3 $a \in \mathbb{R}$

$$\left[\lg(3^x+2) - x\lg 3 - \frac{2}{3^x}\right]e^{ax}$$

$$\longrightarrow \ell \in \mathbb{R} \setminus \{0\}$$

$x \to +\infty$

$$\lg(3^x+2) - \lg 3^x$$

$$\left[\lg\left(1+\frac{2}{3^x}\right) - \cancel{\frac{2}{3^x}}\right] e^{ax}$$

$$\lg(1+t) = \cancel{t} - \frac{t^2}{2} + o(t^2) \qquad t \to 0$$

$$t = \frac{2}{3^x} \qquad x \to +\infty$$

$$\left[-\frac{4}{3^{2x}}\cdot\frac{1}{2} + o\left(\frac{1}{9^x}\right)\right]e^{ax}$$

$$-\frac{2e^{ax}}{9^x} + o\left(\frac{e^{ax}}{9^x}\right)$$

STESSA BASE

$$-2\boxed{\frac{e^{ax}}{e^{x\lg 9}}} + o(\dots)$$

D. 4 $\quad \cos t = 1 - \frac{t^2}{2} + O(t^4)_{t \to 0} \qquad t = 2x$

$$\Big(\underbrace{1 - 2x^2 + O(x^4)}_{y}\Big)^{1/2} - 1$$

$$(1+y)^{\alpha} = 1 + \alpha y + o(y)$$

$$\cancel{1} - x^2 + O(x^4) + o(x^2) \cancel{-1}$$

Oppure oleva7. $\quad \dfrac{\cos 2x - 1}{\sqrt{\cos 2x} + 1} - 1$

etc . . . .

D5 a? $\exists \lim_{0^+} \dfrac{(\tan x)^2 - x^2}{x^a}$

$$\operatorname{tg} x \underset{x \to 0}{=} x + \alpha x^3 + O(x^5)$$

$\sin x = x - \frac{x^3}{6}$

$\alpha \neq 0$ dato per buono

$$\cancel{x^2} + \alpha^2 x^6 + 2\alpha x^4 + O(x^6) \cancel{- x^2}$$

D 6) $\displaystyle\lim_{x\to 0} \frac{\sqrt{e} - (\cos x)^{-(x^{-2})}}{x^2}$

$(1+q)^{1/q} \longrightarrow e$

$q = o(1)$ $(q \to 0)$

$e^{\frac{1}{q}\lg(1+q)}$

$\sqrt{e} - e^{\boxed{-\frac{1}{x^2}\lg\cos x}}$

$-\frac{1}{x^2}\lg\left(1 - \frac{x^2}{2} + \frac{x^4}{24} + O(x^6)\right)$

$-\frac{1}{x^2}\left[-\frac{x^2}{2} + \frac{x^4}{24} + O(x^4)\right]$

$\boxed{\frac{1}{2} + O(x^2)}$

$$\frac{\sqrt{e} - (\cos x)^{-x^{-2}}}{x^2} = \frac{\sqrt{e} - e^{\frac{1}{2} + O(x^2)}}{x^2} =$$

$$= \sqrt{e} \, \frac{1 - e^{O(x^2)}}{x^2}$$

$$-\frac{1}{x^2} \log\Big(\underbrace{1 - \frac{x^2}{2} + \frac{x^4}{24} + O(x^6)}_{t}\Big)$$

devo usare t^2 poiché $O(t)$ contiene t^4

$$\log(1+t) = t - \frac{t^2}{2} + O(t^3)$$

$$-\frac{1}{x^2}\left(-\frac{x^2}{2}+\frac{x^4}{24}-\frac{1}{2}\left(-\frac{x^2}{2}+\frac{x^4}{24}+O(x^6)\right)^2+O(x^6)\right)$$

$$-\frac{1}{2}\frac{x^4}{4}$$

$$\frac{1}{2}-\frac{x^2}{24}+\frac{x^2}{8}+O(x^4)$$

$$1$$

$$f(x)=\sqrt{e}\;\frac{1-e^{\frac{x^2}{12}}}{x^2}=-\frac{\sqrt{e}}{12}\;\frac{e^{\frac{x^2}{12}}-1}{\frac{x^2}{12}}$$

D7 ? $(x+1)^a - x^a - a x^{a-1} \xrightarrow[]{x \to +\infty} 0$

$x^a \left[\left(1 + \frac{1}{x}\right)^a - 1 - \frac{a}{x} \right] =$

$x^a \left[\frac{a(a-1)}{2} \frac{1}{x^2} + o\left(\frac{1}{x^2}\right) \right]$

$(1+t)^a \underset{t \to 0}{=} 1 + at + o(t)$

$(1+t)^a = 1 + at + \frac{a(a-1)}{2} t^2 + o(t^2)$

$\frac{a(a-1)}{2} \cdot x^{a-2} + o(x^{a-2}) \xrightarrow[x \to +\infty]{?} 0$

D8 $\quad \lambda \in \mathbb{R} \quad \underline{\underline{\operatorname{Ozd}(\lambda)}}$

$$1 + x + \frac{x^{2}}{2} + \frac{x^{3}}{6} + O(x^4) - 1 - \lambda x - \frac{\lambda(\lambda-1)}{2}x^2 - \frac{\lambda(\lambda-1)(\lambda-2)}{6}x^3$$

$\underline{\lambda = 1}$ $\quad \operatorname{ord} = 2$

$\lambda \neq 1 \quad \operatorname{ord} = 1$

D 9 Compito a casa

D 10 pol. di Tay di ord 5 in 0 di $\sin(\log(\cos x))$

$\exists !\, p \in \mathbb{R}[x]$ deg $p \le n$:

$$f(x) = p(x) + o(x - x_0)^n$$

$$\lg(\cos x) - \frac{1}{6}\left[\lg(\cos x)\right]^3 + O\left[\lg(\cos x)\right]^5$$

$$\lg\Big(\underbrace{1-\frac{x^2}{2}+\frac{x^4}{24}+O(x^6)}_{t}\Big)-\frac{1}{6}\big[\quad\big]^3+O\big[\quad\big]^5$$

$$t-\boxed{\frac{t^2}{2}}+O(t^3)$$

$$O(x^6)$$

$$-\frac{x^2}{2}+\frac{x^4}{24}-\boxed{\frac{x^4}{8}}$$

$$-\frac{x^2}{2}-\frac{x^4}{12}+\frac{1}{6}\left[\frac{x^2}{2}+\frac{x^4}{12}+O(x^6)\right]^3+O(x^6)$$

$$\boxed{-\frac{x^2}{2}-\frac{x^4}{12}}+o(x^5)$$

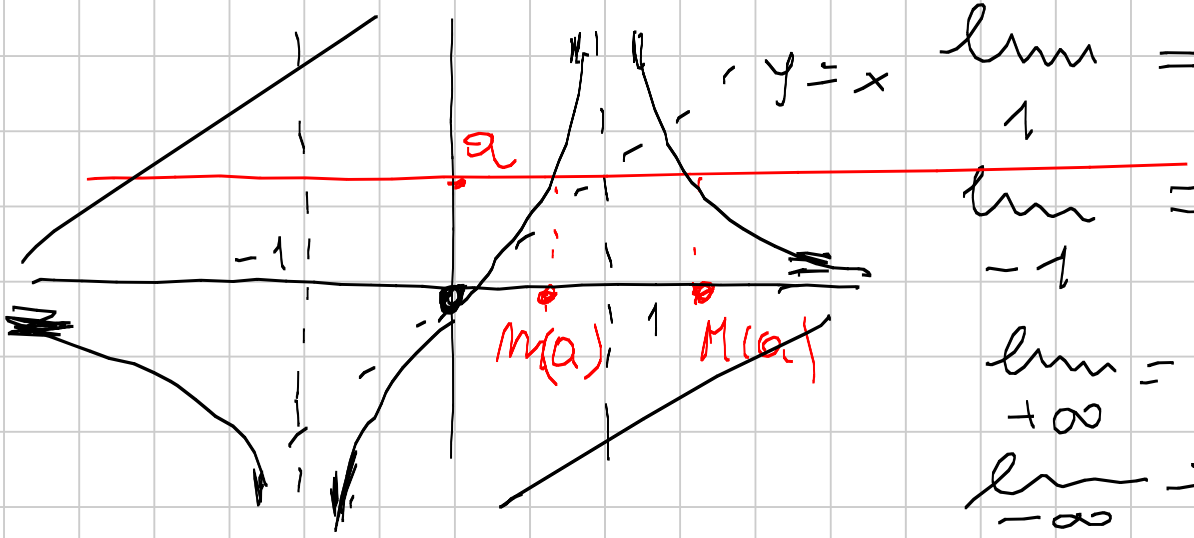
Es. 1. a

DISPARI

$f'(0) = 1$

$f' = -\frac{1+3x^2}{(x^2-1)^3}$

$f'' = 12(x^2+1)x \,/\, (x^2-1)^4$



$\lim_{1} = +\infty$

$\lim_{-1} = -\infty$

$\lim_{+\infty} = 0^+$

$\lim_{-\infty} = 0^-$

$a > 0$ $f(x) = a$ 2 $m(a) < 1 < M(a)$

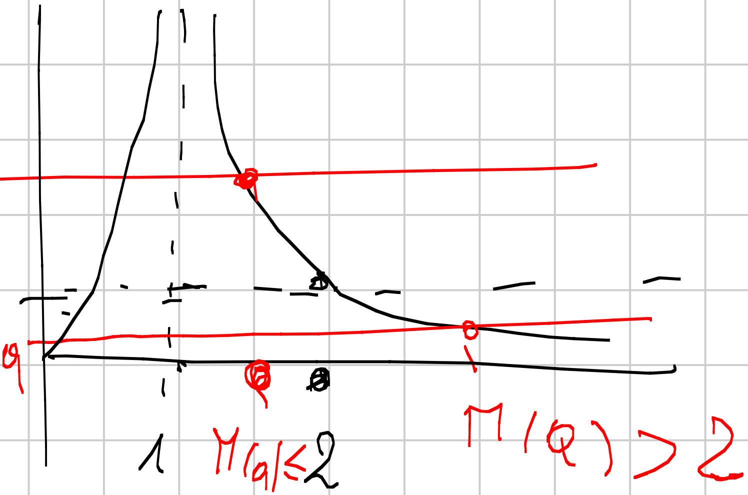
$m(a) < 1 < 2$? $M(a)$

$$\begin{cases} f(x) = a \\ x \leq 2 \end{cases}$$

$x \le 2$ $\qquad M(a) \le 2$

$$\frac{x}{(x^2-1)^2} = a$$

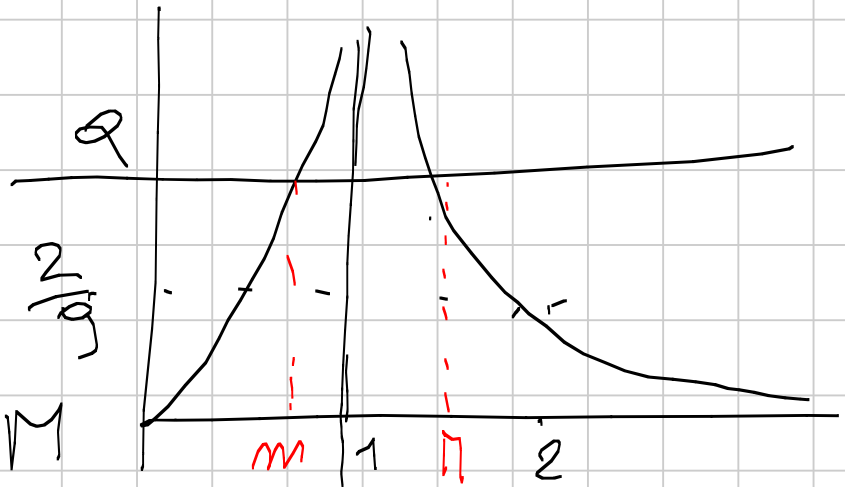
$f(2) = \frac{2}{9}$



$$\begin{cases} 0 < a \le \frac{2}{9} & 1 \\ a \geq \frac{2}{9} & 2 \end{cases}$$

b)

$f|_{(1,+\infty)} = h$ iniettiva $\quad h^{-1}(a) = M$



$f|_{(0,1)} = g$ è inj.

$g^{-1}(a) = m(a)$

PROVARE A FINIRLO A CASA

$$\left(\varphi^{-1}\right)'(Q) = \frac{1}{\varphi'(\varphi^{-1}(Q))}$$

$$\left(\varphi^{-1}\right)'' = \left(\frac{1}{\varphi'(\varphi^{-1}(Q))}\right)' = -\frac{\varphi'' \cdot (\varphi^{-1})'}{\left(\varphi'(\varphi^{-1}(Q))\right)^2}$$