Chapter 2

Networks & Graphs: Basic Measures

Summary
- Graph representations
- Type of Networks
- Degree distribution
- Paths & Connectedness
- Clustering

Reading
- Chapter 2 of Barabasi’s book
The Bridges of Konigsberg

Can one walk across the seven bridges and never cross the same bridge twice?

Euler’s theorem (1735)

- If a graph has more than two nodes of odd degree, there is no path.
- If a graph is connected and has no odd degree nodes, it has at least one path.

Famous Konigsberg Citizens
Immanuel Kant (philosopher, 1724-1804)
Components of a Complex System

Networks or Graphs?

**Network** <nodes, links>
refers to real systems
(www, social network, metabolic network)

**Graph** <vertices, edges>
mathematical representation of a network
(web graph, social graph)

<table>
<thead>
<tr>
<th>Components</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>nodes, vertices</td>
<td>N</td>
</tr>
<tr>
<td>edges, links</td>
<td>L</td>
</tr>
<tr>
<td>network, graph</td>
<td>(N,L)</td>
</tr>
</tbody>
</table>
The choice of the proper network representation determines our ability to use network theory successfully. In some cases there is a unique, unambiguous representation. In other cases, the representation is by no means unique.

The way we assign the links between a group of individuals will determine the nature of the question we can study.
Proper representations (examples)

If you connect individuals that work with each other, you will explore the *professional network*.

If you connect those that have a romantic and sexual relationship, you will be exploring the *sexual networks*.

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The structure of adolescent romantic and sexual networks

http://researchnews.osu.edu/archive/chainspix.htm
If you connect individuals based on their first name (e.g., all Peters connected to each other), you will be exploring what?

It is a network, nevertheless.
Directedness

**Undirected graphs**
Links: undirected (*symmetrical*)

**Directed graphs** *(DiGraphs)*
Links: directed (*arcs*).

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**Examples of Undirected links**
- Co-authorship links
- Actor network
- Protein interactions

**Example of Directed links**
- URLs on the www
- Phone calls
- Metabolic reactions
<table>
<thead>
<tr>
<th>NETWORK</th>
<th>NODES</th>
<th>LINKS</th>
<th>DIRECTED UNDIRECTED</th>
<th>N</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internet</td>
<td>Routers</td>
<td>Internet connections</td>
<td>Undirected</td>
<td>192,244</td>
<td>609,066</td>
</tr>
<tr>
<td>WWW</td>
<td>Webpages</td>
<td>Links</td>
<td>Directed</td>
<td>325,729</td>
<td>1,497,134</td>
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<td>Power Grid</td>
<td>Power plants, transformers</td>
<td>Cables</td>
<td>Undirected</td>
<td>4,941</td>
<td>6,594</td>
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<td>Subscribers</td>
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<td>Scientists</td>
<td>Co-authorship</td>
<td>Undirected</td>
<td>23,133</td>
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<tr>
<td>Actor Network</td>
<td>Actors</td>
<td>Co-acting</td>
<td>Undirected</td>
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<td>29,397,908</td>
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<tr>
<td>Citation Network</td>
<td>Paper</td>
<td>Citations</td>
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<td>4,689,479</td>
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<tr>
<td>E. Coli Metabolism</td>
<td>Metabolites</td>
<td>Chemical reactions</td>
<td>Directed</td>
<td>1,039</td>
<td>5,862</td>
</tr>
<tr>
<td>Protein Interactions</td>
<td>Proteins</td>
<td>Binding interactions</td>
<td>Undirected</td>
<td>2,018</td>
<td>2,930</td>
</tr>
</tbody>
</table>

Reference Networks
Degree, Average Degree, Degree Distribution
Node Degree

**Undirected graphs**
the number of links connected to the node

\[ k_A = 1 \]
\[ k_B = 4 \]

**Directed graphs (DiGraphs)**
we can define an in-degree and out-degree.
The (total) degree is the sum of in- and out-degree.

\[ k_{in}^C = 2 \]
\[ k_{out}^C = 1 \]
\[ k_C = 3 \]

Source: a node with \( k_{in} = 0 \);
Sink: a node with \( k_{out} = 0 \).
A Bit of Statistics

Four key quantities characterize a sample of $N$ values $x_1, \ldots, x_n$

Average (mean):

$$\langle x \rangle = \frac{x_1 + x_2 + \ldots + x_n}{N} = \frac{1}{N} \sum_{i=1}^{n} x_i$$

The $n^{th}$ moment:

$$\langle x^n \rangle = \frac{x_1^n + x_2^n + \ldots + x_n^n}{N} = \frac{1}{N} \sum_{i=1}^{N} x_i^n$$

The Standard deviation:

$$\sigma_x = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \langle x \rangle)^2}$$

The Distribution of $x$:

$$p_x = \frac{1}{N} \sum_{i} \delta_{x,x_i}$$

where $p_x$ follows

$$\sum_{x} p_x = 1 \left( \int p_x \, dx = 1 \right)$$
Average Degree

Undirected graphs
N - the number of nodes in the graph

\[ \langle k \rangle = \frac{1}{N} \sum_{i=1}^{N} k_i \]
\[ \langle k \rangle = \frac{2L}{N} \]

Directed graphs (DiGraphs)

\[ \langle k^{in} \rangle = \frac{1}{N} \sum_{i=1}^{N} k_i^{in} \]
\[ \langle k^{out} \rangle = \frac{1}{N} \sum_{i=1}^{N} k_i^{out} \]

\[ \langle k^{in} \rangle = \langle k^{out} \rangle \]
\[ \langle k \rangle = \frac{L}{N} \]
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Reference Networks: Average Degree
Degree Distribution

$P(k)$: probability that a randomly chosen node has degree $k$

$N_k = \# \text{ nodes with degree } k$

$P(k) = \frac{N_k}{N} \rightarrow \text{ plot}$
Degree Distribution (cont’d)

**Discrete Representation:** $p_k$ is the probability that a node has degree $k$.

**Continuum Description:** $p(k)$ is the pdf of the degrees, where

$$\int_{k_1}^{k_2} p(k) \, dk$$

represents the probability that a node’s degree is between $k_1$ and $k_2$.

**Normalization condition:**

$$\sum_{a}^{\infty} p_k = 1 \quad \int_{K_{\text{min}}}^{\infty} p(k) \, dk = 1$$

where $K_{\text{min}}$ is the minimal degree in the network.
Adjacency matrix

Undirected graphs

\[ A_{ij} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \]

\[ k_i = \sum_{j=1}^{N} A_{ij} \]
\[ k_j = \sum_{i=1}^{N} A_{ij} \]
\[ A_{ij} = A_{ji} \]
\[ A_{ii} = 0 \]

\[ L = \frac{1}{2} \sum_{i=1}^{N} k_i = \frac{1}{2} \sum_{ij} A_{ij} \]

Directed graphs (DiGraphs)

\[ A_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} \]

\[ k^\text{in}_i = \sum_{j=1}^{N} A_{ij} \]
\[ k^\text{out}_j = \sum_{i=1}^{N} A_{ij} \]
\[ A_{ij} \neq A_{ji} \]
\[ A_{ii} = 0 \]

\[ L = \sum_{i=1}^{N} k^\text{in}_i = \sum_{j=1}^{N} k^\text{out}_j = \sum_{i,j} A_{ij} \]
Paths and Connectedness
A path is a sequence of nodes in which each node is adjacent to the next one.

\[ P_{i_0,i_n} \text{ of length } n \text{ between nodes } i_0 \text{ and } i_n \text{ is an ordered collection of } n+1 \text{ nodes and } n \text{ links} \]

\[ P_n = \{i_0, i_1, i_2, \ldots, i_n\} \]

\[ P_n = \{(i_0, i_1), (i_1, i_2), (i_2, i_3), \ldots, (i_{n-1}, i_n)\} \]
Distance in a Graph

**Undirected graphs**
The distance (shortest path, geodesic path) between two nodes is defined as the number of edges along the shortest path connecting them.

*If the two nodes are disconnected, the distance is infinity.*

**Directed graphs (DiGraphs)**
Each path needs to follow the direction of the arrows.

Thus in a digraph the distance from node A to B (on an AB path) is generally different from the distance from node B to A (on a BCA path).
Number of paths between two nodes

\( N_{ij} \), number of paths between any two nodes \( i \) and \( j \):

**Length \( n=1 \):**
If there is a link between \( i \) and \( j \), then \( A_{ij} = 1 \) and \( A_{ij} = 0 \) otherwise.

**Length \( n=2 \):**
If there is a path of length two between \( i \) and \( j \), then \( A_{ik} \cdot A_{kj} = 1 \), and \( A_{ik} \cdot A_{kj} = 0 \) otherwise.

The number of paths of length 2:

\[
N^{(2)}_{ij} = \sum_{k=1}^{N} A_{ik} A_{kj} = [A^2]_{ij}
\]

**Length \( n \):**
In general, if there is a path of length \( n \) between \( i \) and \( j \), then \( A_{ik} \ldots A_{ij} = 1 \) and \( A_{ik} \ldots A_{ij} = 0 \) otherwise.

The number of paths of length \( n \) between \( i \) and \( j \) is (*)

\[
N^{(n)}_{ij} = [A^n]_{ij}
\]

(*) Holds for both directed and undirected networks.
Finding Distances: BFS

Distance between node 0 and node 4:

1. Start at 0.
Finding Distances: BFS

Distance between node 0 and node 4:

1. Start at 0.
2. Find the nodes adjacent to 1. Mark them as at distance 1. Put them in a queue.
Finding Distances: BFS

Distance between node 0 and node 4:

1. Start at 0.
2. Find the nodes adjacent to 1. Mark them as at distance 1. Put them in a queue.
3. Take the first node out of the queue. Find the unmarked nodes adjacent to it in the graph. Mark them with the label of 2. Put them in the queue.
Finding Distances: BFS

Distance between node 0 and node 4:

Repeat until you find node 4 or there are no more nodes in the queue.

The distance between 0 and 4 is the label of 4 or, if 4 does not have a label, infinity.
Diameter and Average distance

**Diameter** ($d_{\text{max}}$):

the maximum distance between any pair of nodes in the graph.

**Average path length/distance**, $<d>$, for a connected graph:

$$
<d> \equiv \frac{1}{2L_{\text{max}}} \sum_{i,j \neq i} d_{ij}
$$

where $d_{ij}$ is the distance from node $i$ to node $j$.

In an undirected graph $d_{ij} = d_{ji}$, so we only need to count them once:

$$
<d> \equiv \frac{1}{L_{\text{max}}} \sum_{i,j>i} d_{ij}
$$
Paths: a summary

Shortest Path
The path with the shortest length between two nodes (distance).
Paths: a summary

Diameter
The longest shortest path in a graph.

$l_{1→4} = 3$
Paths: a summary

Average Path Length
The average of the shortest paths for all pairs of nodes.

\[
\frac{(l_{1\rightarrow 2} + l_{1\rightarrow 3} + l_{1\rightarrow 4} + \\
+ l_{1\rightarrow 5} + l_{2\rightarrow 3} + l_{2\rightarrow 4} + \\
+ l_{2\rightarrow 5} + l_{3\rightarrow 4} + l_{3\rightarrow 5} + \\
+ l_{4\rightarrow 5})}{10} = 1.6
\]
Paths: a summary

**Cycle**
A path with the same start and end node.

**Self-Avoiding Path**
A path that does not intersect itself.
Paths: a summary

**Eulerian Path/Cycle**
A path that traverses each link exactly once.

**Hamiltonian Path/Cycle**
A path that visits each node exactly once.
Connectivity of undirected graphs

**Connected (undirected) graph**: any two vertices can be joined by a path.

A **disconnected graph** is made up by two or more connected components.

**Bridge**: if we erase it, the graph becomes disconnected. Example (A,F)

Largest Component: **Giant Component**
The rest: **Isolates**
Connectivity of undirected graphs

The adjacency matrix of a network with several components can be written in a block-diagonal form, so that nonzero elements are confined to squares, with all other elements being zero.
Connectivity of directed graphs

**Strongly connected directed graph (SCC):** has a path from each node to every other node and vice versa (e.g. AB path and BA path).

**Weakly connected directed graph (WCC):** it is connected if we disregard the edge directions.

Strongly connected components can be identified, but not every node is part of a nontrivial strongly connected component.

In-component: nodes that can reach the scc.

Out-component: nodes that can be reached from the scc.
Network Density
A graph with degree \( L = L_{\text{max}} \)
is called a complete graph, and its average degree is

\[ <k> = N - 1 \]

The maximum number of links an undirected network of \( N \) nodes can have is:

\[
L_{\text{max}} = \binom{N}{2} = \frac{N(N - 1)}{2}
\]

What about directed networks?
Network Density

Ratio of existing edges over possible ones.

Complete graph
\[ d(G) = \frac{L}{L_{max}} \]

Examples

Low Density

High Density

Complete graph
\[ d(G) = 1 \]

Sparse graph
\[ d(G) = 2/3 \]
Most networks observed in real systems are **sparse**

$L \ll L_{\text{max}}$

$\langle k \rangle \ll N-1$

$d(G) \ll 1$

### Sparse Adjacency matrix

<table>
<thead>
<tr>
<th>Network Type</th>
<th>$N$</th>
<th>$L$</th>
<th>$L_{\text{max}}$</th>
<th>$\langle k \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>WWW (ND Sample)</td>
<td>325,729</td>
<td>$1.4 \times 10^6$</td>
<td>$10^{12}$</td>
<td>4.51</td>
</tr>
<tr>
<td>Protein (S. Cerevisiae)</td>
<td>1,870</td>
<td>4,470</td>
<td>$10^7$</td>
<td>2.39</td>
</tr>
<tr>
<td>Coauthorship (Math)</td>
<td>70,975</td>
<td>$2 \times 10^5$</td>
<td>$3 \times 10^{10}$</td>
<td>3.9</td>
</tr>
<tr>
<td>Movie Actors</td>
<td>212,250</td>
<td>$6 \times 10^6$</td>
<td>$1.8 \times 10^{13}$</td>
<td>28.78</td>
</tr>
</tbody>
</table>

(Source: Albert, Barabasi, RMP2002)
Clustering Coefficient
Clustering Coefficient

How “clustered” is my network?

Global Clustering coefficient
- Triangles and triplets
- C in [0,1]

\[ C = \frac{3 \times \text{number of triangles}}{\text{number of all triplets}} \]

Clustering Coefficient

What fraction of your neighbors are connected?

Local Clustering coefficient
- Node $i$ with degree $k_i$
- $C_i$ in $[0,1]$

\[
C_i = \frac{2e_i}{k_i(k_i - 1)}
\]

Clustering Coefficient

What fraction of your neighbors are connected?

Local Clustering coefficient
- Node i with degree $k_i$
- $C_i$ in [0,1]

\[
\langle C \rangle = \frac{13}{42} \approx 0.310
\]

\[
C = \frac{3}{8} = 0.375
\]

\[
C_i = \frac{2e_i}{k_i(k_i - 1)}
\]

Clustering Coefficient

What fraction of your neighbors are connected on average?

Average Clustering coefficient
- Average of local clustering coefficients
- $C$ in $[0,1]$

\[
\bar{C} = \frac{1}{n} \sum_{i=1}^{n} C_i
\]

$\bar{C} \approx 0.31$

Bipartite Networks
Bipartite Graphs

Bipartite graph (or bigraph)

A graph whose nodes can be divided into two disjoint sets $U$ and $V$ such that every link connects a node in $U$ to one in $V$.

**Examples**
- Hollywood actor network
- Collaboration networks
- Disease network (diseasome)

**Projection**
Two nodes of the same class are connected by a (weighted) edge if they share at least a common neighbor.
Gene network (projection)

DISEASOME

PHENOME

GENOME

Disease network (projection)

Gene - DiseNetwork ase Network

Goh, Cusick, Valle, Childs, Vidal & Barabási, PNAS (2007)
Ingredient-Flavor Bipartite Network

Y.-Y. Ahn, S. E. Ahnert, J. P. Bagrow, A.-L. Barabási
Summarizing...
Central quantities in Network Science

Degree Distribution

\[ P(k) \]

Path length

\[ <d> \]

Clustering Coefficient

\[ C_i = \frac{2e_i}{k_i(k_i - 1)} \]
Type of graphs

Directedness

Undirected graph

\[ A_{ij} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \]

\[ A_{ii} = 0 \quad A_{ij} = A_{ji} \]

\[ L = \frac{1}{2} \sum_{i,j=1}^{N} A_{ij} \quad <k> = \frac{2L}{N} \]

Actor network, protein-protein interactions

Directed graph

WWW, citation networks

\[ A_{ij} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \]

\[ A_{ii} = 0 \quad A_{ij} \neq A_{ji} \]

\[ L = \sum_{i,j=1}^{N} A_{ij} \quad <k> = \frac{L}{N} \]
**Type of graphs**

**Weightedness**

**Unweighted graph**

\[ A_{ij} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \]

\[ A_{ii} = 0 \quad A_{ij} = A_{ji} \]

\[ L = \frac{1}{2} \sum_{i,j=1}^{N} A_{ij} \quad <k> = \frac{2L}{N} \]

**Weighted graph**

\[ A_{ij} = \begin{pmatrix} 0 & 2 & 0.5 & 0 \\ 2 & 0 & 1 & 4 \\ 0.5 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{pmatrix} \]

\[ A_{ii} = 0 \quad A_{ij} = A_{ji} \]

\[ L = \frac{1}{2} \sum_{i,j=1}^{N} \text{nonzero}(A_{ij}) \quad <k> = \frac{2L}{N} \]

*protein-protein interactions, WWW*

*Call Graph, metabolic networks*
Type of graphs

**Loops & Multigraphs**

**Self Interactions**

\[ A_{ij} = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \]

\[ L = \frac{1}{2} \sum_{i,j=1}^{N} A_{ij} + \sum_{i=1}^{N} A_{ii} \]

\[ A_{ii} \neq 0 \quad A_{ij} = A_{ji} \]

protein-protein interactions, WWW

**Multigraph (undirected)**

\[ A_{ij} = \begin{pmatrix} 0 & 2 & 1 & 0 \\ 2 & 0 & 1 & 3 \\ 1 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 \end{pmatrix} \]

\[ L = \frac{1}{2} \sum_{i,j=1}^{N} \text{nonzero}(A_{ij}) \quad < k > = \frac{2L}{N} \]

Social Network, Collaboration Network
Real Networks can have multiple characteristics

<table>
<thead>
<tr>
<th>Network</th>
<th>Directed</th>
<th>Weighted</th>
<th>Multigraph</th>
<th>Self-loops</th>
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<tbody>
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<td>WWW</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Protein interactions</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Collaboration network</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Mobile phone calls</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Facebook Friendship</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
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Case Study: Protein-Protein Interaction Network
Case study

Protein-protein interaction

Undirected network
N=2,018 proteins as nodes
L=2,930 binding interactions as links.
Average degree $<k>$=2.90.

Not connected: 185 components
Largest (giant component) 1,647 nodes
Case study

Protein-protein interaction

$p_k$ is the probability that a node has degree $k$

$N_k = \# \text{ nodes with degree } k$

$p_k = N_k / N$
Case study

Protein-protein interaction

Path length distribution

\[ d_{\text{max}} = 14 \]

\[ \langle d \rangle = 5.61 \]
Case study

Protein-protein interaction

Clustering coefficient vs. node degree

Average Clustering Coeff.

\[ <C> = 0.12 \]
Chapter 2

Conclusion

Take Away Messages
1. Semantic shapes graph topology
2. Network properties can be measured
3. Degree distribution
4. Paths & Connectivity
5. Clustering Coefficient

Suggested Readings
- Chapter 2 of Barabasi’s book
- Chapter 2 of Kleinberg’s book

What’s Next
Chapter 3: Random Networks