RAPPRESENTAZIONI NUMERI IN MACCHINA

$$1.234 = 1\,10^{0} + 2\,10^{-1} + 3\,10^{-2} + 4\,10^{-3}$$

1 2 3 4 cifre del numero.

$$(1.011)_2 = 1 \cdot 2^{0} + 0 \cdot 2^{-1} + 1\,2^{-2} + 1\,2^{-3}$$

β base di numerazione. cifre $0 \le d_i \le \beta - 1$

$$(0.012)_{10} = 0 \cdot 10^{0} + 0 \cdot 10^{-1} + 1 \cdot 10^{-2} + 2 \cdot 10^{-3}$$

$$\|$$

$$(0.12)_{10} \times 10^{-1}$$

$$\|$$

$$(x) \cdot 10^{p} \qquad |x| < 1$$

Teorema (rappresentazione dei numeri)

Sia $x \in \mathbb{R}$, $x \neq 0$ e sia $B \in \mathbb{N}$ $B > 1$.

Allora $\exists !$ $p \in \mathbb{Z}$ (esponente) e $\{d_i\}_{i \in \mathbb{N}}$

con $d_1 \neq 0$ e d_i non definitivamente uguali a $B-1$ tale che

$$x = \text{sign}(x)\, B^p \cdot \sum_{i=1}^{+\infty} d_i \cdot B^{-i}$$

↑ mantissa $|x| \leq 1$

Rappresentazione normalizzata in virgola mobile del numero x

(floating point)

$d_1 \neq 0$

$$(0.01)_{10} = \cancel{(0.01)\,10^{0}}$$

$$(0.1)\cdot 10^{-1}$$

$$(0.\overline{9})_{10} = 0.\underbrace{99\ldots9}\; 10^{0}$$

$$(0.\overline{1})_2 = \underbrace{0.11\ldots1}$$

$$0.\overline{9} = 0.9\ldots9 = \sum_{i=1}^{+\infty} 9 \cdot 10^{-i}$$

$$= 9 \sum_{i=1}^{+\infty} 10^{-i} = \lim_{k\to+\infty} \sum_{i=1}^{k} 10^{-i}$$

$$10^{-1} + 10^{-2} + \cdots + 10^{-k} = \frac{1}{10} + \left(\frac{1}{10}\right)^2 + \cdots + \left(\frac{1}{10}\right)^k$$

$$a + a^2 + \dots + a^k \qquad \left(a = \frac{1}{10}\right)$$

$$(1-a)\cdot\left(a+a^2+\dots+a^k\right) = a + a^2 + \dots + a^k - \left(a^2 + a^3 + \dots + a^{k+1}\right)$$

$$= a - a^{k+1} = a\cdot\left(1-a^k\right)$$

$$a + a^2 + \dots + a^k = \frac{a\cdot\left(1-a^k\right)}{1-a} \qquad (a \neq 1)$$

$$\lim_{k \to +\infty} a + \dots + a^k = \lim_{k \to +\infty} \frac{a \cdot (1 - a^k)}{1 - a} \qquad a = \frac{1}{10}$$

$$= \lim_{k \to +\infty} \frac{a - a^{k+1}}{1 - a} = \frac{a}{1-a}$$

$a^{k+1} \to 0$

$$\sum_{i=1}^{+\infty} 10^{-i} = \frac{\frac{1}{10}}{1 - \frac{1}{10}} = \frac{\frac{1}{10}}{\frac{9}{10}} = \frac{1}{9}$$

$$0.\overline{9} = 0.9\ldots9 = \sum_{i=1}^{+\infty} 9\cdot 10^{-i}$$

$$= 9\cdot\sum_{i=1}^{+\infty} 10^{-i} = 9\cdot\frac{1}{9} = 1$$

$$(0.1)_{10}\cdot 10^{1}$$

$$\cancel{(0.\overline{9})\cdot 10^{0}}$$

$x \neq 0$

$\cancel{0 = \beta^{p} \sum_{|z|}^{+\infty} d_1 \cdot \beta^{-u}}$

$d_1 \neq 0$

$x \in \mathbb{R}$

$x = a + ib$ $\quad i^2 = -1 \quad a, b \in \mathbb{R}$

$\updownarrow$

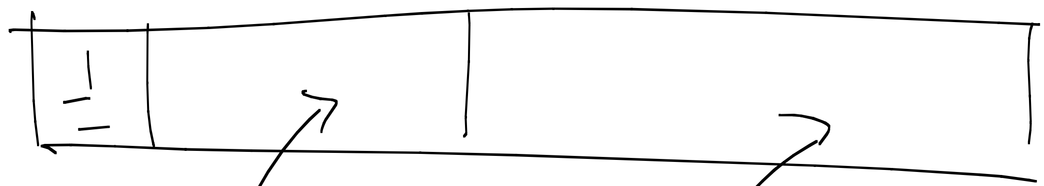
$(a, b) \in \mathbb{R}^2$

$$X = \mathrm{sign}(x) \cdot \beta^{p} \cdot \sum_{i=1}^{+\infty} d_i \cdot \beta^{-i} \qquad (X \neq 0)$$

$d_1 \neq 0$

$d_i \not\equiv \beta - 1$

$$X \longleftrightarrow \mathrm{sign}(x), \; p \text{ esponente}, \; \{d_i\}_{i \in \mathbb{N}} \text{ cifre mantissa}$$



p esponente

cifre della mantissa

$$\tilde{x} = \pm B^p \cdot \sum_{i=1}^{t} d_i \cdot B^{-i}$$

$$-m \le p \le M$$

$x \in \mathbb{R} \rightsquigarrow \tilde{x}$ di macchina