Model Free Control

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Outline

- ✓Introduction
- ✓On-policy Vs Off-Policy
- ✓On-policy Monte-Carlo
- ✓On-policy TD learning (SARSA)
- ✓Off-policy TD (Q-learning)

Introduction

Today's focus

- ✓ Last lecture
  - ✓ Model-free prediction
  - ✓ Estimate the value function of an unknown MDP
- ✓ Today's lecture
  - ✓ Model-free control
  - ✓ Optimise the value function of an unknown MDP

Model Free Control – Where to find it

- ✓Elevator
- ✓Robot walking
- ✓Vehicle Steering
- ✓Bioreactor
- ✓Molecule engineering
- ✓Robocup Soccer
- ✓Quake
- ✓Portfolio management
- ✓Protein Folding
- ✓Game of Go

For most of these problems, either:

- ✓MDP model is unknown, but experience can be sampled
- ✓MDP model is known, but is too big to use, except by samples

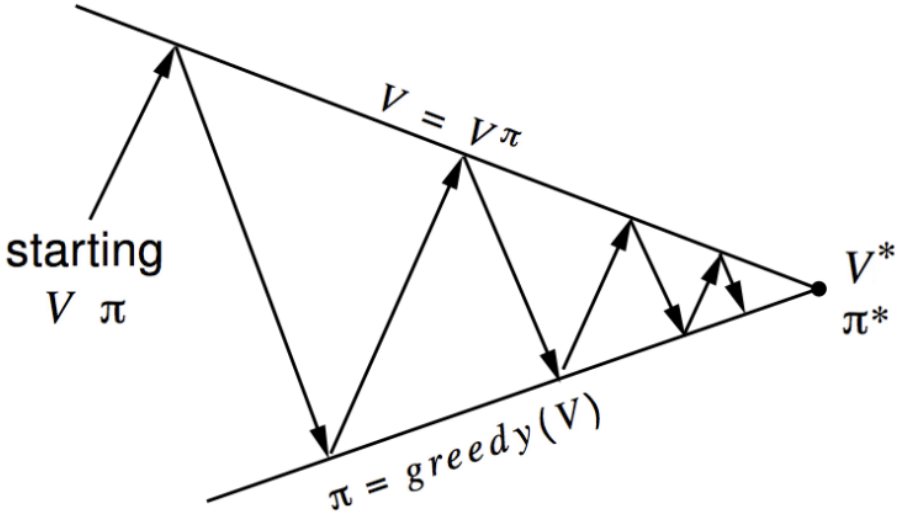
Model-free control can solve these problems

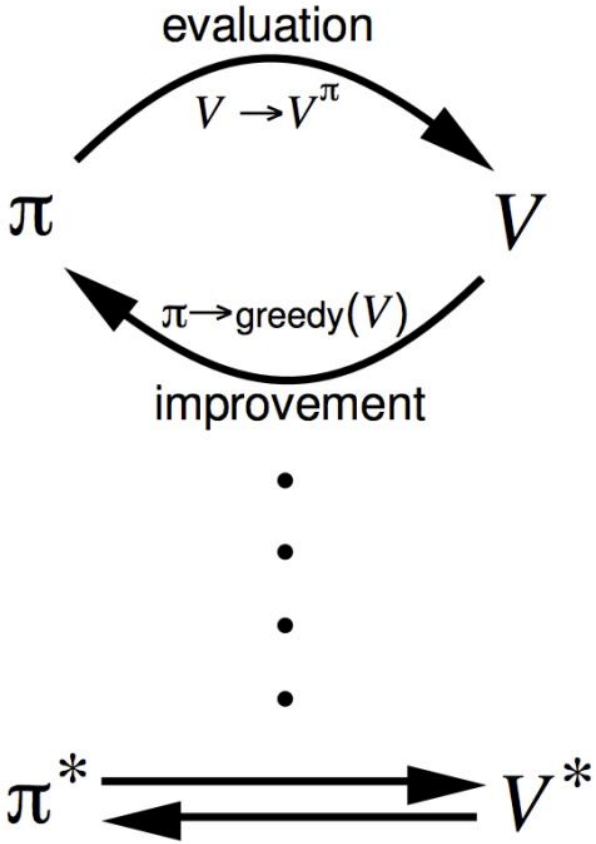
On-policy & Off-policy Learning

- ✓ On-policy learning
  - ✓ *Learn on the job*
  - ✓ Learn about policy π from experience sampled from π
- ✓ Off-policy learning
  - ✓ *Look over someone's shoulder*
  - ✓ Learn about policy π from experience sampled from μ

On-policy MC

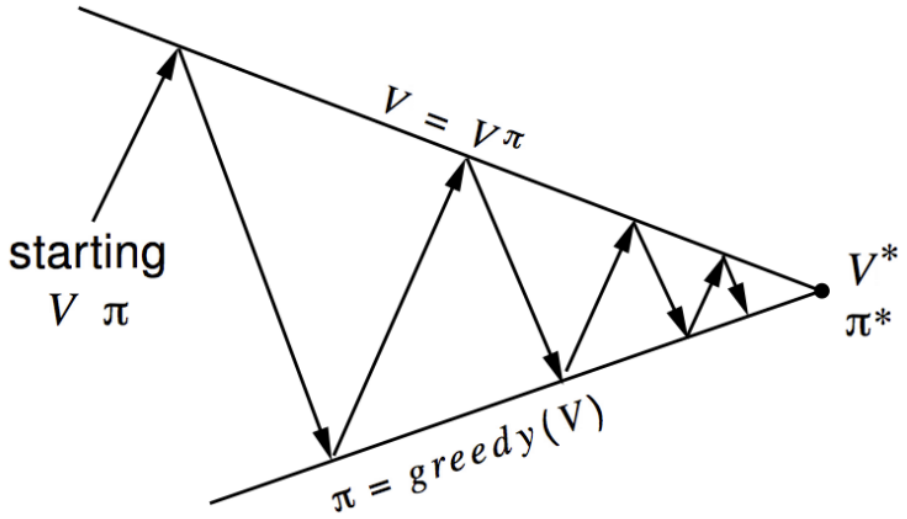
Generalized Policy Iteration (Lecture 3)





- ✓Policy evaluation - Estimate v_π
  - ✓Any policy evaluation
- ✓Policy improvement - Generate $\pi' \geq \pi$
  - ✓Any policy improvement algorithm

Generalized Policy Iteration with On-policy MC



- ✓Policy evaluation - Monte-Carlo policy evaluation, $V = v_\pi$?
- ✓Policy improvement - Generate greedy policy improvement?

Model-Free Policy Iteration Using Action-Value Function

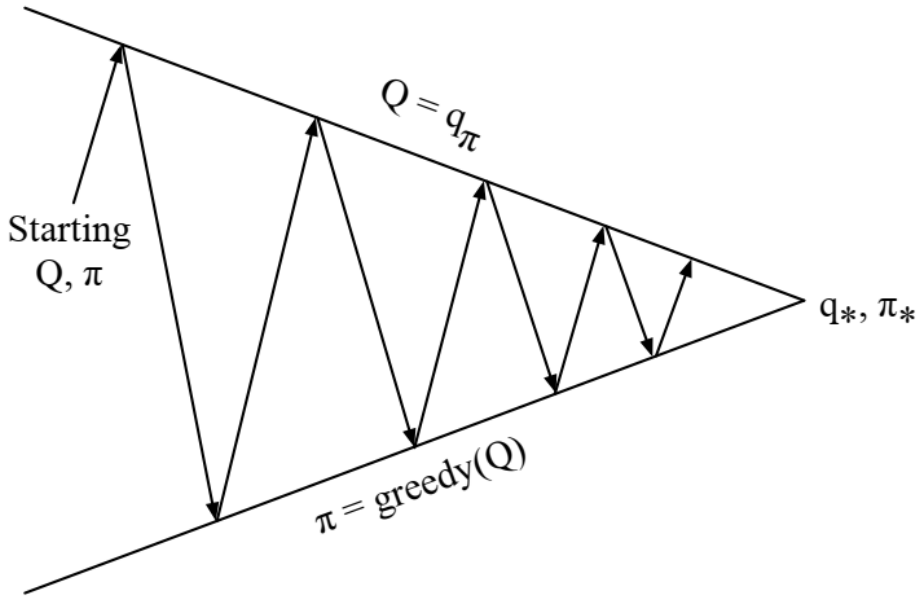
- ✓ Greedy policy improvement over $V(s)$ requires model of MDP

$$\pi'(s) = \arg\max_{a \in \mathcal{A}} \mathcal{R}_s^a + P_{ss'}^a V(s')$$

- ✓ Greedy policy improvement over $\mathrm{Q}(s, a)$ is model-free

$$\pi'(s) = \arg\max_{a \in \mathcal{A}} Q(s, a)$$

Generalized Policy Iteration with Action-Value Function



✓Policy evaluation - Monte-Carlo policy evaluation, $Q = q_\pi$

✓Policy improvement - Generate Greedy policy improvement?

Example of Greedy Action Selection

"Behind one door is tenure - behind the other is flipping burgers at McDonald's."

There are two doors in front of you.

- ✓You open the left door and get reward 0 – V (left) = 0
- ✓You open the right door and get reward +1 - V (right) = +1
- ✓You open the right door and get reward +3 - V (right) = +2
- ✓You open the right door and get reward +2 - V (right) = +2

…

- ✓Are you sure you've chosen the best door?

ϵ-greedy Exploration

- ✓ Simplest idea for ensuring continual exploration
- ✓ All m actions are tried with non-zero probability
  - ✓ With probability $1 - \epsilon$ choose the greedy action
  - ✓ With probability ϵ choose an action at random

$$\pi(a|s) = \begin{cases} \epsilon/m + (1 - \epsilon) & \text{if } a^* = \arg\max_{a \in \mathcal{A}} Q(s, a) \\ \epsilon/m & \text{otherwise} \end{cases}$$

ϵ-greedy Policy Improvement

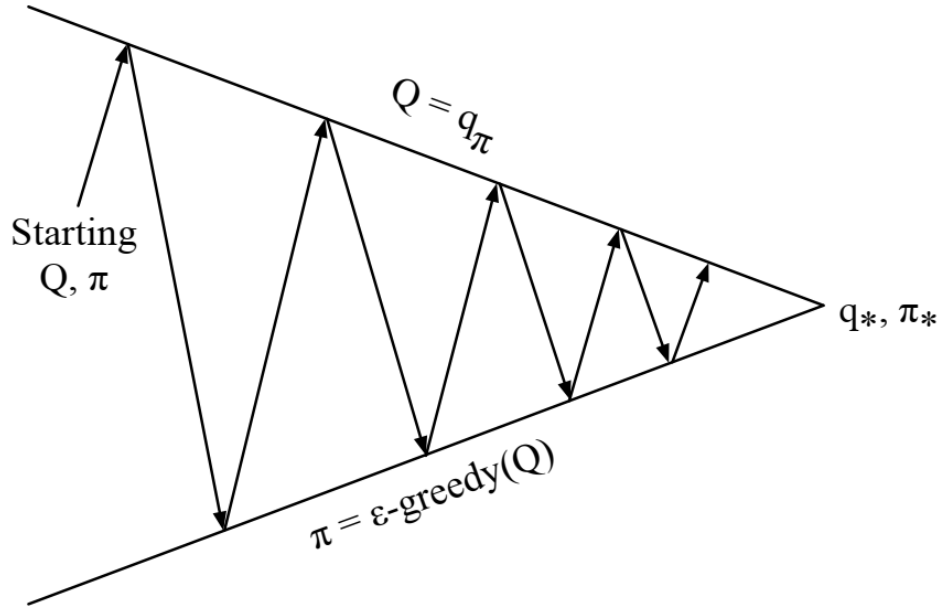
Theorem

For any ϵ-greedy policy π, the ϵ-greedy policy π' with respect to q_π is an improvement, $v_{\pi'}(s) \geq v_\pi(s)$

$$
\begin{aligned}
q_\pi\big(s, \pi'(s)\big) &= \sum_{a \in \mathcal{A}} \pi'(a|s) q_\pi(s,a) \\
&= {}^{\epsilon}/_{m} \sum_{a \in \mathcal{A}} q_\pi(s,a) + (1-\epsilon) \max_{a \in \mathcal{A}} q_\pi(s,a) \\
&\geq {}^{\epsilon}/_{m} \sum_{a \in \mathcal{A}} q_\pi(s,a) + (1-\epsilon) \sum_{a} \frac{\pi(a|s) - {}^{\epsilon}/_{m}}{1-\epsilon} q_\pi(s,a) \\
&= \sum_{a \in \mathcal{A}} \pi(a|s) q_\pi(s,a) = v_\pi(s)
\end{aligned}
$$

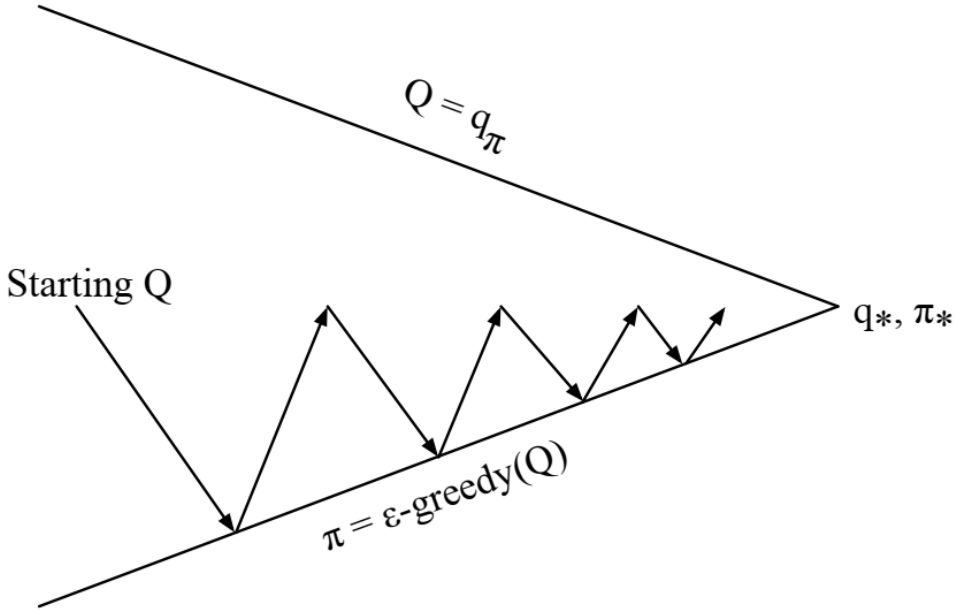
Therefore from policy improvement theorem $v_{\pi'}(s) \geq v_\pi(s)$

Monte-Carlo Policy Iteration



- ✓Policy evaluation - Monte-Carlo policy evaluation, $Q = q_\pi$
- ✓Policy improvement - ϵ-greedy policy improvement

Monte-Carlo Control



Every Episode

✓Policy evaluation - Monte-Carlo policy evaluation, $Q \approx q_\pi$

✓Policy improvement - ϵ-greedy policy improvement

Greedy in the Limit with Infinite Exploration (GLIE)

Definition (Greedy in the Limit with Infinite Exploration - GLIE)

All state-action pairs are explored infinitely many times

$$\lim_{k\to\infty} N_k(s,a) = \infty$$

The policy converges on a greedy policy

$$\lim_{k\to\infty} \pi_k(a|s) = \mathbf{1}\left(a; \arg\max_{a'\in\mathcal{A}} Q_k(s,a')\right)$$

ϵ-greedy is GLIE if ϵ reduces to zero at $\epsilon_k = \frac{1}{k}$

GLIE Monte Carlo Control

- ✓Sample k-th episode using π: $\{S_1, A_1, R_2, \dots, S_T\} \sim \pi$
- ✓For each state S_t and action A_t in the episode:

$$N(S_t, A_t) \leftarrow N(S_t, A_t) + 1$$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \frac{1}{N(S_t, A_t)}\big(G_t - Q(S_t, A_t)\big)$$

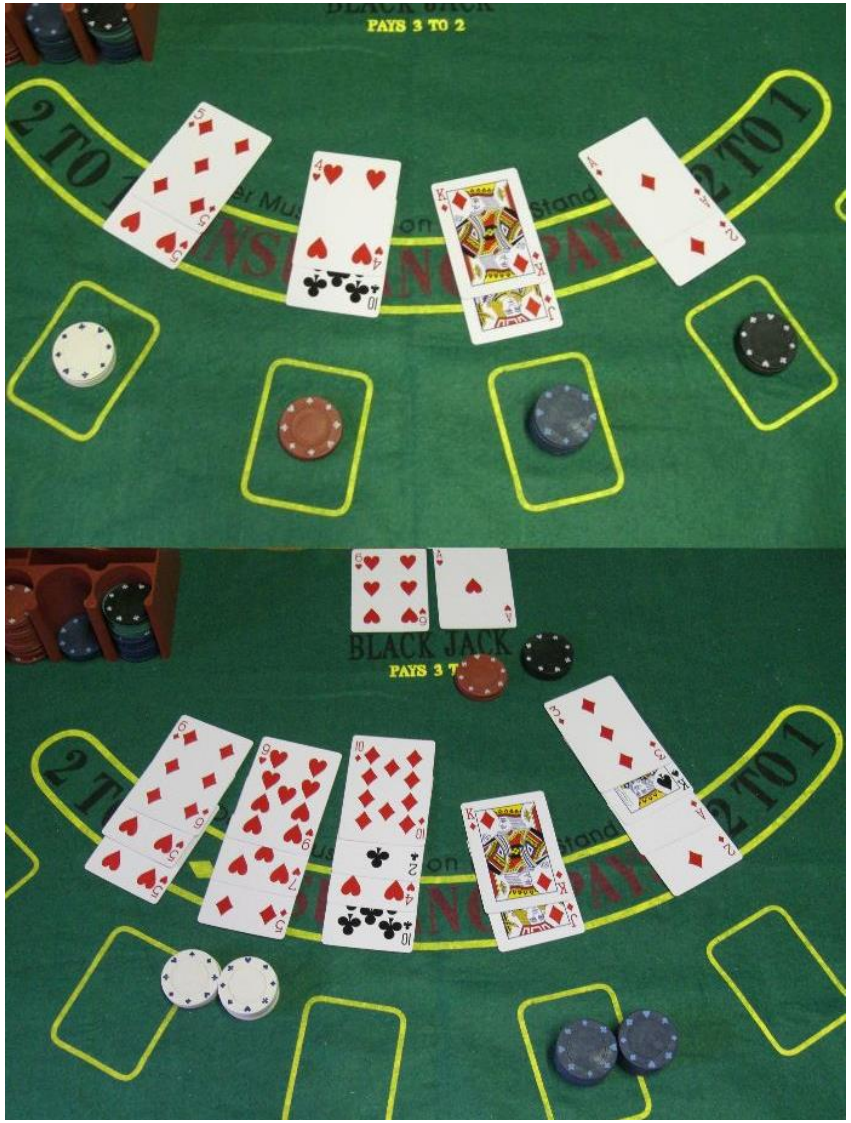
- ✓Improve policy based on new action-value function

$$\epsilon \leftarrow \frac{1}{k}$$

$$\pi \leftarrow \epsilon - \text{greedy}(Q)$$

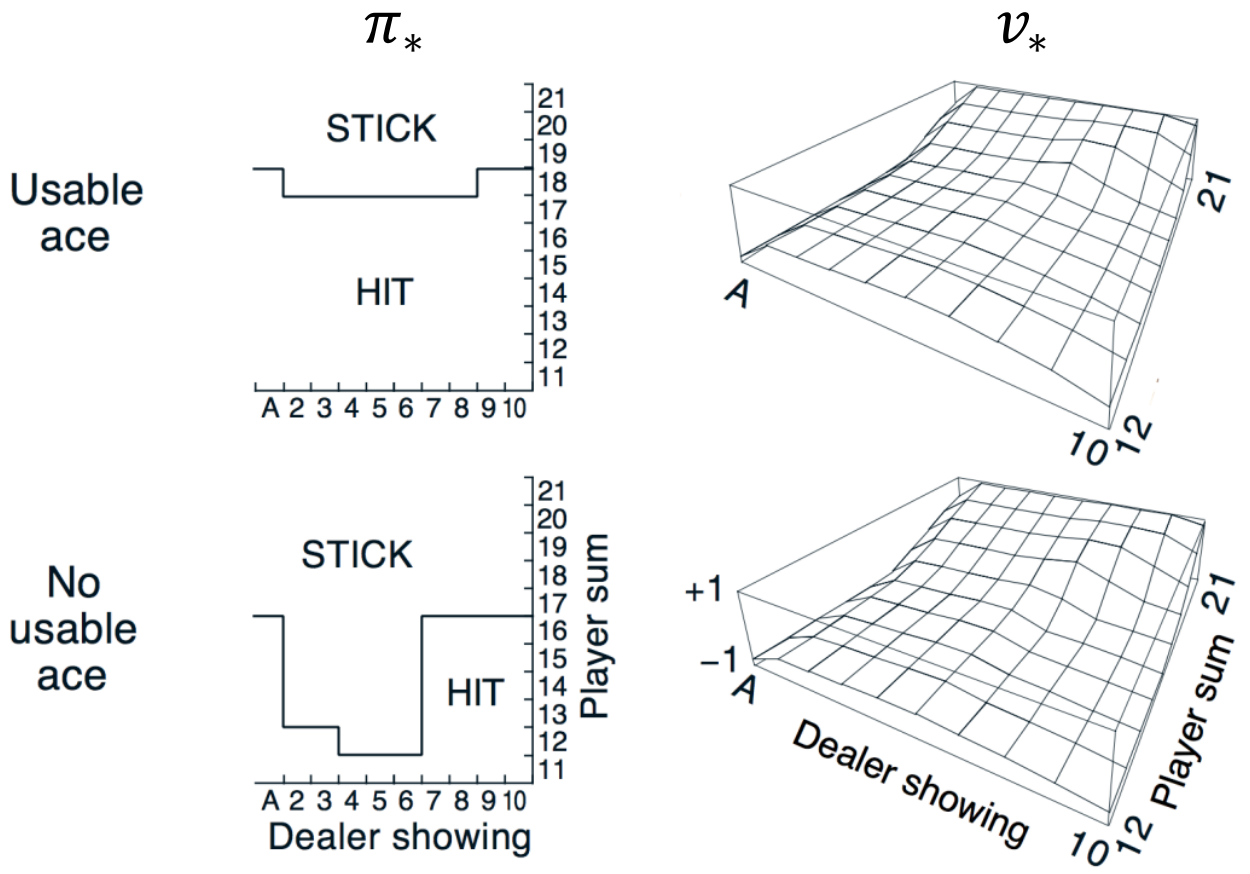
Theorem

GLIE Monte-Carlo control converges to the optimal action-value function $Q(s, a) \rightarrow q_*(s, a)$



Blackjack Example

Blackjack – MC Control

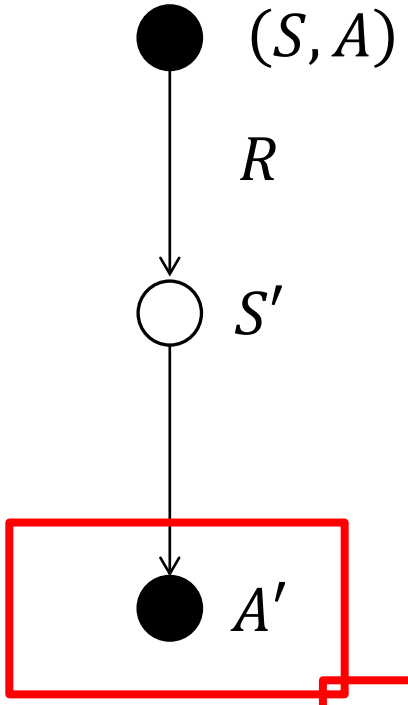


On-Policy TD Control

MC Vs TD Control

- ✓TD learning has several advantages over MC
  - ✓Lower variance
  - ✓Online
  - ✓Incomplete sequences
- ✓Straightforward intuition - Use TD instead of MC in our control loop
  - ✓Apply TD to $Q(s, a)$
  - ✓Use ϵ-greedy policy improvement
  - ✓Update every time-step

Updating Action-Value Functions with SARSA



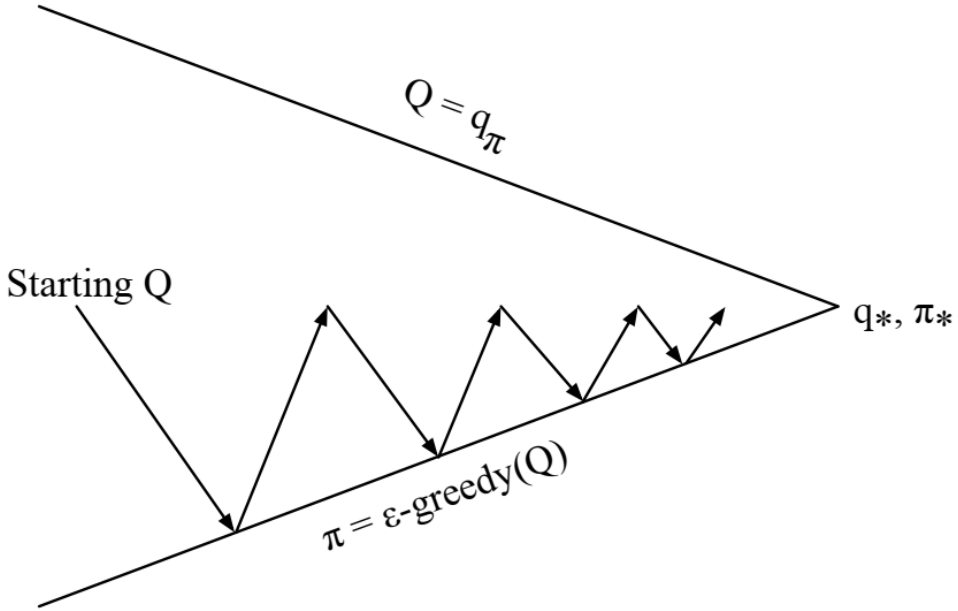
Expected SARSA

$$q_*(s,a) = \mathcal{R}_s^a + \gamma \sum_{a' \in \mathcal{A}} \pi(a'|s') q_*(s', a')$$

We sample also future action A'
(instead of leveraging policy to compute expectation)

$$Q(S,A) \leftarrow Q(S,A) + \alpha(R + \gamma Q(S', A') - Q(S,A))$$

On-Policy Control with SARSA



Every time-step

✓Policy evaluation - SARSA, $Q \approx q_\pi$

✓Policy improvement - ϵ-greedy policy improvement

SARSA Algorithm for On-Policy Control

Sarsa (on-policy TD control) for estimating $Q \approx q_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

Loop for each episode:
    Initialize S
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Loop for each step of episode:
        Take action A, observe R, S'
        Choose A' from S' using policy derived from Q (e.g., ε-greedy)
        $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma Q(S', A') - Q(S, A)\big]$
        $S \leftarrow S'$; $A \leftarrow A'$;
    until S is terminal

Convergence of SARSA

Theorem

Sarsa converges to the optimal action-value function ($Q(s,a) \rightarrow q_*(s,a)$) under the following conditions:

- GLIE sequence of policies $\pi_t(a|s)$
- Robbins-Monro sequence of step-sizes α_t

$$\sum_{t=1}^{\infty} \alpha_t = \infty$$

$$\sum_{t=1} \alpha_t^2 < \infty$$

Time for TD Demo

https://cs.stanford.edu/people/karpathy/reinforcejs/gridworld_td.html

SARSA(λ)

n-step SARSA

✓Consider the following n -step returns for $n = 1,2, \dots, \infty$

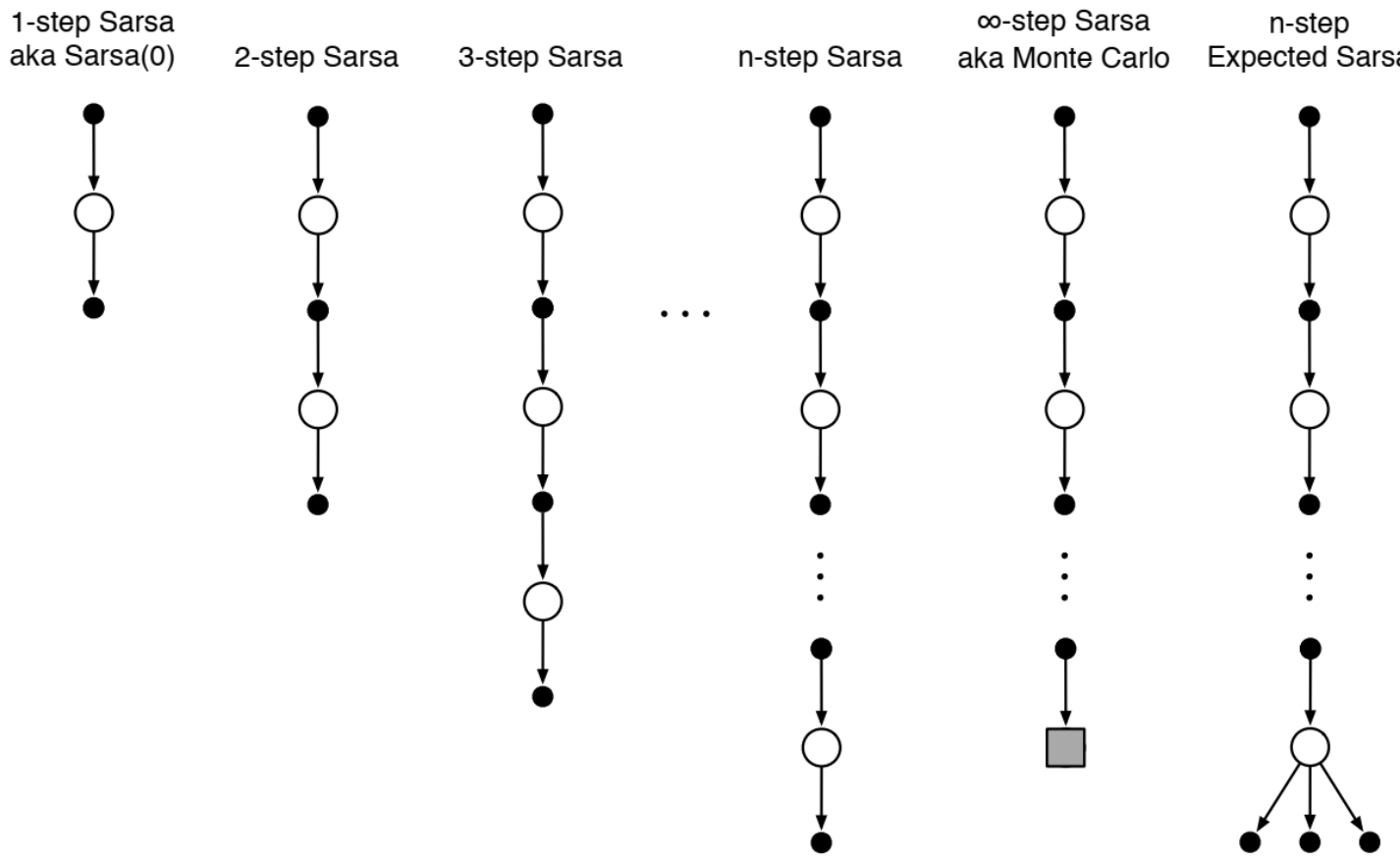
$$n = 1 \quad \text{(SARSA)} \quad q_t^{(1)} = R_{t+1} + \gamma Q(S_{t+1}, A_{t+1})$$
$$n = 2 \qquad q_t^{(2)} = R_{t+1} + \gamma R_{t+2} + \gamma Q(S_{t+2})$$
$$\dots$$
$$n = \infty \quad \text{(MC)} \quad q_t^{(\infty)} = R_{t+1} + \gamma R_{t+2} + \cdots + \gamma^{T-1} R_T$$

✓Define the n-step Q-return

$$q_t^{(n)} = R_{t+1} + \gamma R_{t+2} + \cdots + \gamma^{n-1} R_{t+n} + \gamma^n Q(S_{t+n}, A_{t+n})$$

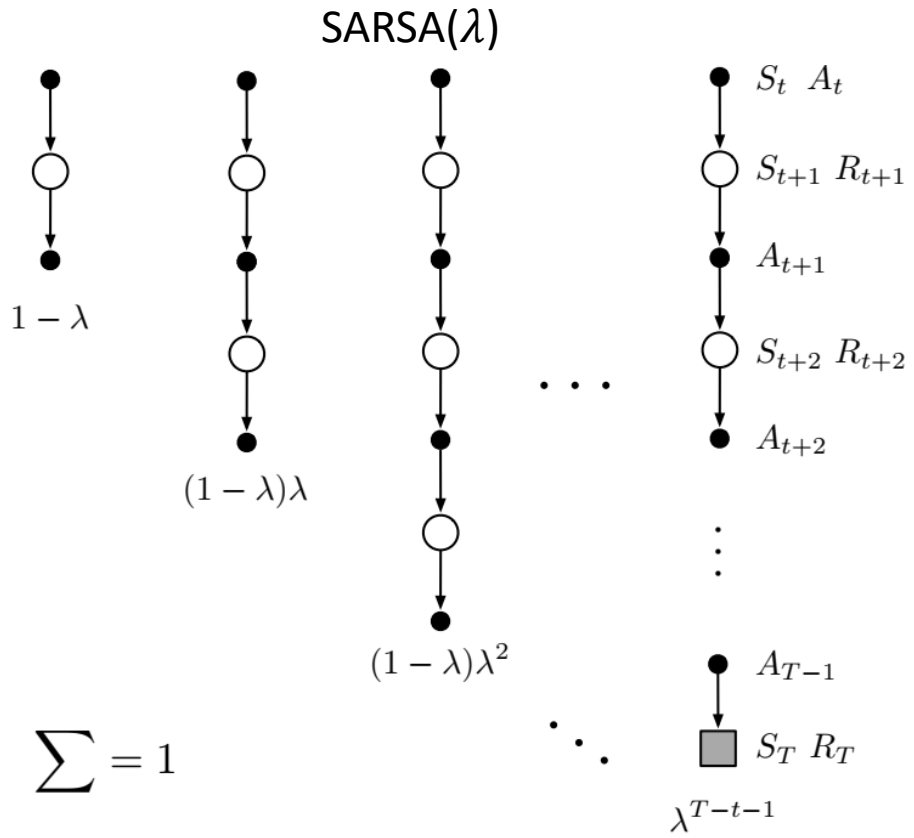
✓n-step SARSA updates $Q(S, A)$ towards the n-step Q-return

$$Q(S, A) \leftarrow Q(S, A) + \alpha \left(q_t^{(n)} - Q(S, A) \right)$$



SARSA backups

SARSA(λ) - Forward View



- ✓The q^λ return combines all n-step Q-returns $q_t^{(n)}$
- ✓Using weight $(1-\lambda)\lambda^{n-1}$

$$q_t^\lambda = (1-\lambda)\sum_{n=1}^{\infty} \lambda^{n-1} q_t^{(n)}$$

- ✓Forward SARSA update

$$Q(S,A) \leftarrow Q(S,A) + \alpha\left(q_t^\lambda - Q(S,A)\right)$$

SARSA(λ) - Backward View

- ✓ The return of eligibility traces
- ✓ SARSA(λ) needs one eligibility trace for each state-action pair

$$E_0(s,a) = 0$$

$$E_t(s,a) = \gamma\lambda E_{t-1}(s,a) + \mathbf{1}(S_t, A_t; s, a)$$

- ✓ $Q(s,a)$ is updated for every state s and action a in proportion to TD-error δ_t and eligibility trace $E_t(s,a)$

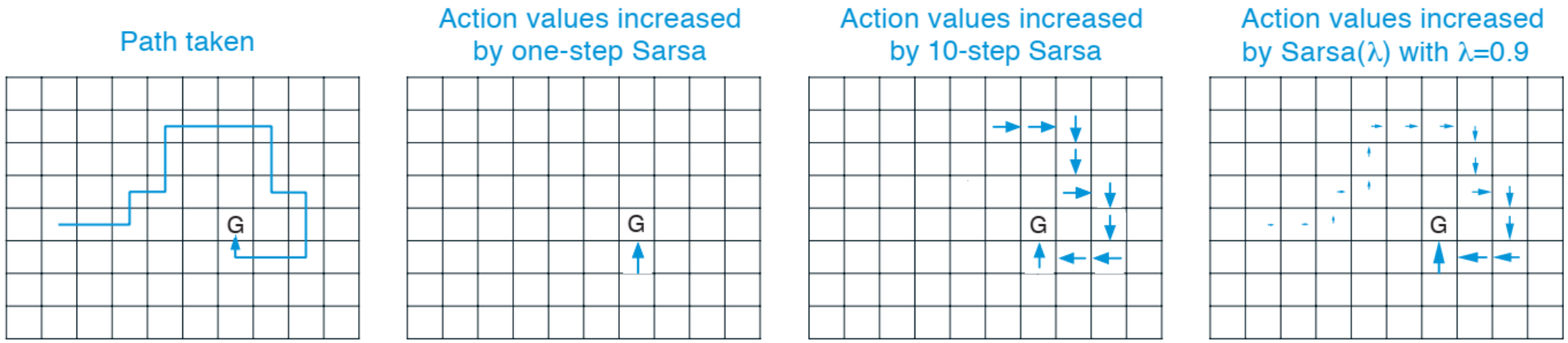
$$\delta_t = R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)$$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\delta_t E_t(s,a)$$

SARSA(λ) Algorithm

Initialize $Q(s, a)$ arbitrarily, for all $s \in \mathcal{S}, a \in \mathcal{A}(s)$
Repeat (for each episode):
  $E(s, a) = 0$, for all $s \in \mathcal{S}, a \in \mathcal{A}(s)$
  Initialize S, A
  Repeat (for each step of episode):
    Take action A, observe R, S'
    Choose A' from S' using policy derived from Q (e.g., ε-greedy)
    $\delta \leftarrow R + \gamma Q(S', A') - Q(S, A)$
    $E(S, A) \leftarrow E(S, A) + 1$
    For all $s \in \mathcal{S}, a \in \mathcal{A}(s)$:
      $Q(s, a) \leftarrow Q(s, a) + \alpha \delta E(s, a)$
      $E(s, a) \leftarrow \gamma \lambda E(s, a)$
    $S \leftarrow S'$; $A \leftarrow A'$
  until S is terminal

SARSA(λ) on Gridworld



Off-policy TD Learning

Off-Policy Learning

- ✓ Evaluate target policy $\pi(a|s)$ to compute $v_\pi(s)$ or $q_\pi(s,a)$
- ✓ While following behaviour policy $\mu(a|s)$

$$\{S_1, A_1, R_2, \dots, S_T\} \sim \mu$$

- ✓ Why is this important?
  - ✓ Learn from imitation (humans, other agents,…)
  - ✓ Re-use experience generated from old policies $\pi_1, \pi_2, \dots, \pi_{t-1}$
  - ✓ Learn about optimal policy while following exploratory policy
  - ✓ Learn about multiple policies while following one policy

(Repetita?) Importance Sampling

Estimate the expectation leveraging an external (importance) distribution

$$\mathbb{E}_{X\sim P}[f(X)] = \sum P(X)f(X)$$

$$= \sum Q(X)\frac{P(X)}{Q(X)}f(X)$$

$$= \mathbb{E}_{X\sim Q}\left[\frac{P(X)}{Q(X)}f(X)\right]$$

Draw samples from importance distribution $Q(X)$ rather than from $P(X)$

Assign weights such that the empirical expectation (on $Q(X)$ samples) matches the expectation under $P(X)$

Importance Sampling for Off-Policy Monte Carlo

- ✓ Use returns generated from μ to evaluate π
- ✓ Weight return G_t according to similarity between policies
- ✓ Multiply importance sampling corrections along whole episode

$$G_t^{\pi/\mu} = \frac{\pi(A_t|S_t)}{\mu(A_t|S_t)} \frac{\pi(A_{t+1}|S_{t+1})}{\mu(A_{t+1}|S_{t+1})} \cdots \frac{\pi(A_T|S_T)}{\mu(A_T|S_T)} G_t$$

- ✓ Update value towards corrected return

$$V(S_t) \leftarrow V(S_t) + \alpha(G_t^{\pi/\mu} - V(S_t))$$

- ✓ Importance sampling can dramatically increase variance

Importance Sampling for Off-Policy TD

- ✓ Use TD targets generated from μ to evaluate π
- ✓ Weight TD targets $R + \gamma V(S')$ by importance sampling
- ✓ Only need a single importance sampling correction

$$V(S_t) \leftarrow V(S_t) + \alpha \left(\frac{\pi(A_t|S_t)}{\mu(A_t|S_t)} (R_{t+1} + \gamma V(S_{t+1}) - V(S_t) \right)$$

- ✓ Much lower variance than MC
- ✓ Policies only need to be similar over a single step

Q-Learning

Off-policy learning of action-values $Q(s, a)$

✓No importance sampling is required

✓Next action is chosen using behaviour policy $A_{t+1} \sim \mu(\cdot\,|S_t)$

✓But we consider alternative successor action $A' \sim \pi(\cdot\,|S_t)$

✓And update $Q(S_t, A_t)$ towards value of alternative action

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\big(R_{t+1} + \gamma Q(S_{t+1}, A') - Q(S_t, A_t)\big)$$

Off-policy Control by Q-Learning

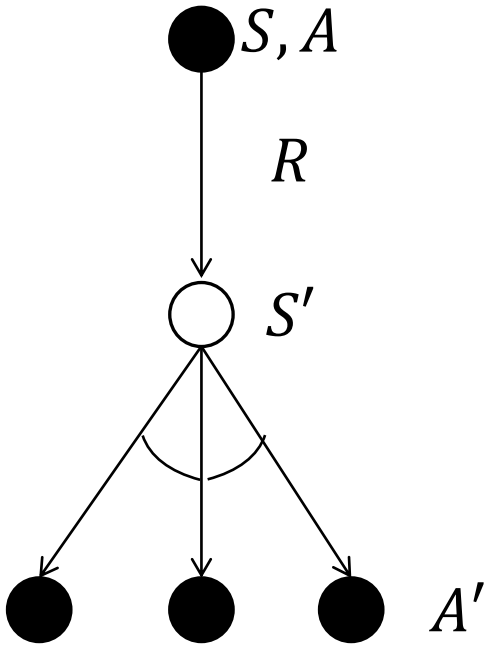
- ✓Allow both behaviour and target policies to improve
- ✓The target policy π is greedy w.r.t. $Q(S_t, A_t)$

$$\pi(S_{t+1}) = \arg\max_{a'} Q(S_{t+1}, a')$$

- ✓The behaviour policy μ is ϵ-greedy w.r.t. $Q(s, a)$
- ✓The Q-learning target then simplifies to

$$\begin{aligned} R_{t+1} + \gamma Q(S_{t+1}, A') &= R_{t+1} + \gamma Q\left(S_{t+1}, \arg\max_{a'} Q(S_{t+1}, a')\right) \\ &= R_{t+1} + \max_{a'} \gamma Q(S_{t+1}, a') \end{aligned}$$

Q-Learning Control Algorithm



Theorem

Q-learning control converges to the optimal action-value function, $Q(s,a) \rightarrow q_*(s,a)$

$$Q(S,A) \leftarrow Q(S,A) + \alpha\left(R + \max_{a'} \gamma Q(S',a') - Q(S,A)\right)$$

Q-Learning Algorithm for Off-policy Control

Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$

Repeat (for each episode):

  Initialize S

  Repeat (for each step of episode):

    Choose A from S using policy derived from Q (e.g., ε-greedy)

    Take action A, observe R, S'

    $Q(S,A) \leftarrow Q(S,A) + \alpha\left[R + \gamma \max_a Q(S',a) - Q(S,A)\right]$

    $S \leftarrow S'$;

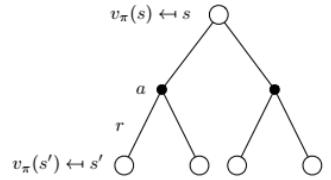
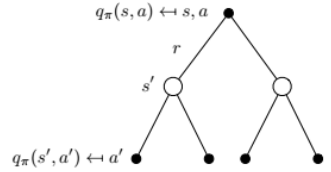
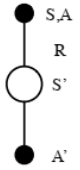
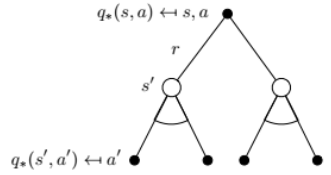
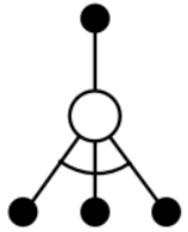
  until S is terminal

Q-learning & Exploration Demo

https://www.aslanides.io/aixijs/demo.html

Wrap-up

Dynamic Programming Vs Temporal Difference Learning

	Full Backup (DP)	*Sample Backup (TD)*
Bellman Expectation Equation for $v_\pi(s)$	 Iterative Policy Evaluation	 TD Learning
Bellman Expectation Equation for $q_\pi(s,a)$	 Q-Policy Iteration	 Sarsa
Bellman Optimality Equation for $q_*(s,a)$	 Q-Value Iteration	 Q-Learning

Take (stay) home messages

- ✓ Model-Free control leverages action-value function
  - ✓ Greedy policy improvement does not need MDP
  - ✓ Generalized policy iteration
- ✓ Need to maintain sufficient exploration (ϵ-greedy)
- ✓ Off-policy control
  - ✓ Learning value function of a target policy from data generated by a different behaviour policy
  - ✓ Importance sampling to match the expectations of two policies
- ✓ TD control
  - ✓ On-policy: SARSA(λ)
  - ✓ Off-policy: Q-learning

Next Lecture

Value-function approximation

- ✓ Leave aside tabular environments
- ✓ Estimate value function with function approximation
- ✓ Linear models & neural networks
- ✓ MC & TD with Stochastic Gradient
- ✓ Experience replay buffers