Wavelets

INTELLIGENT SYSTEMS FOR PATTERN RECOGNITION (ISPR)

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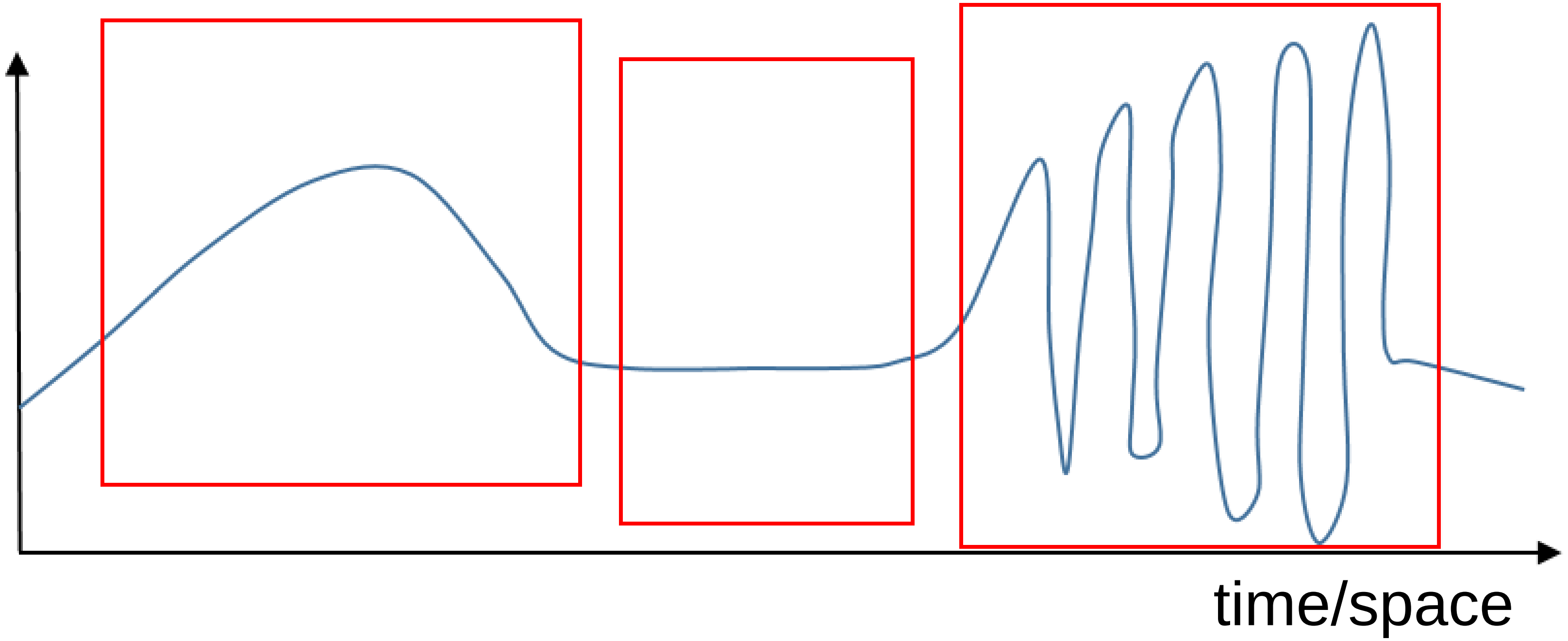
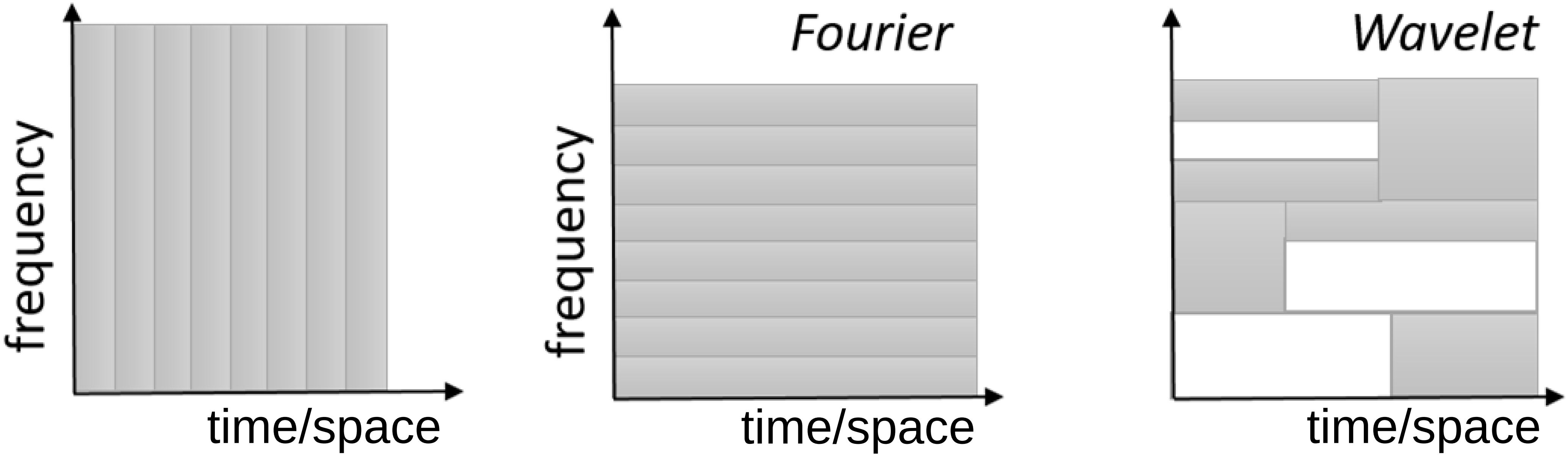
Limitations of DFT

Sometimes we might need localized frequencies rather than global frequency analysis



Graphical Intuition



Split signal in frequency bands only if they exist in specific time-intervals or portion of the space

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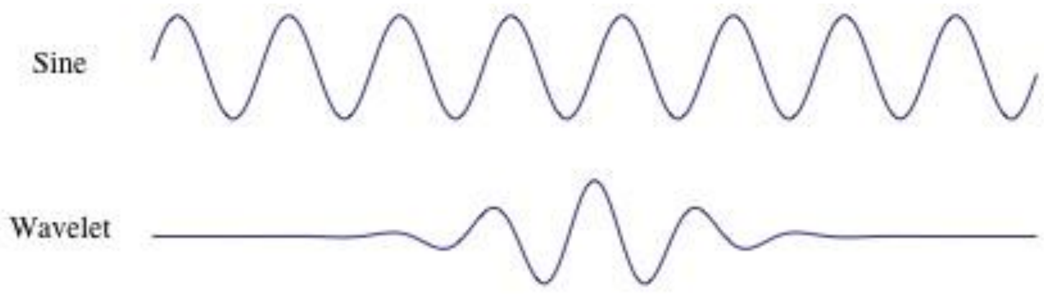
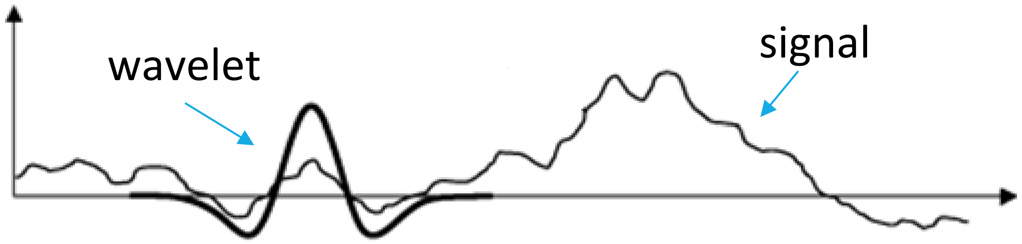
How Does it Work?



Basis function upon with to decompose the signal in Fourier transform

Basis function upon with to decompose the signal in wavelet tranform



Local match of wavelet and signal → Shift wavelet in time/space

Convolution

1. Scale and shift original signal
2. Compare signal to a wavelet
3. Compute a coefficient of similarity

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Wavelets

Split the signal using an orthonormal basis generated by translation and dilation of a mother wavelet

$$\sum_t \boldsymbol{x}(t)\, \Psi_{j,k}(t)$$

Terms k and j regulate scaling and shifting of the wavelet

$$\Psi_{j,k}(t) = \frac{1}{\sqrt{2^k}} \Psi((t-j)/2^k)$$

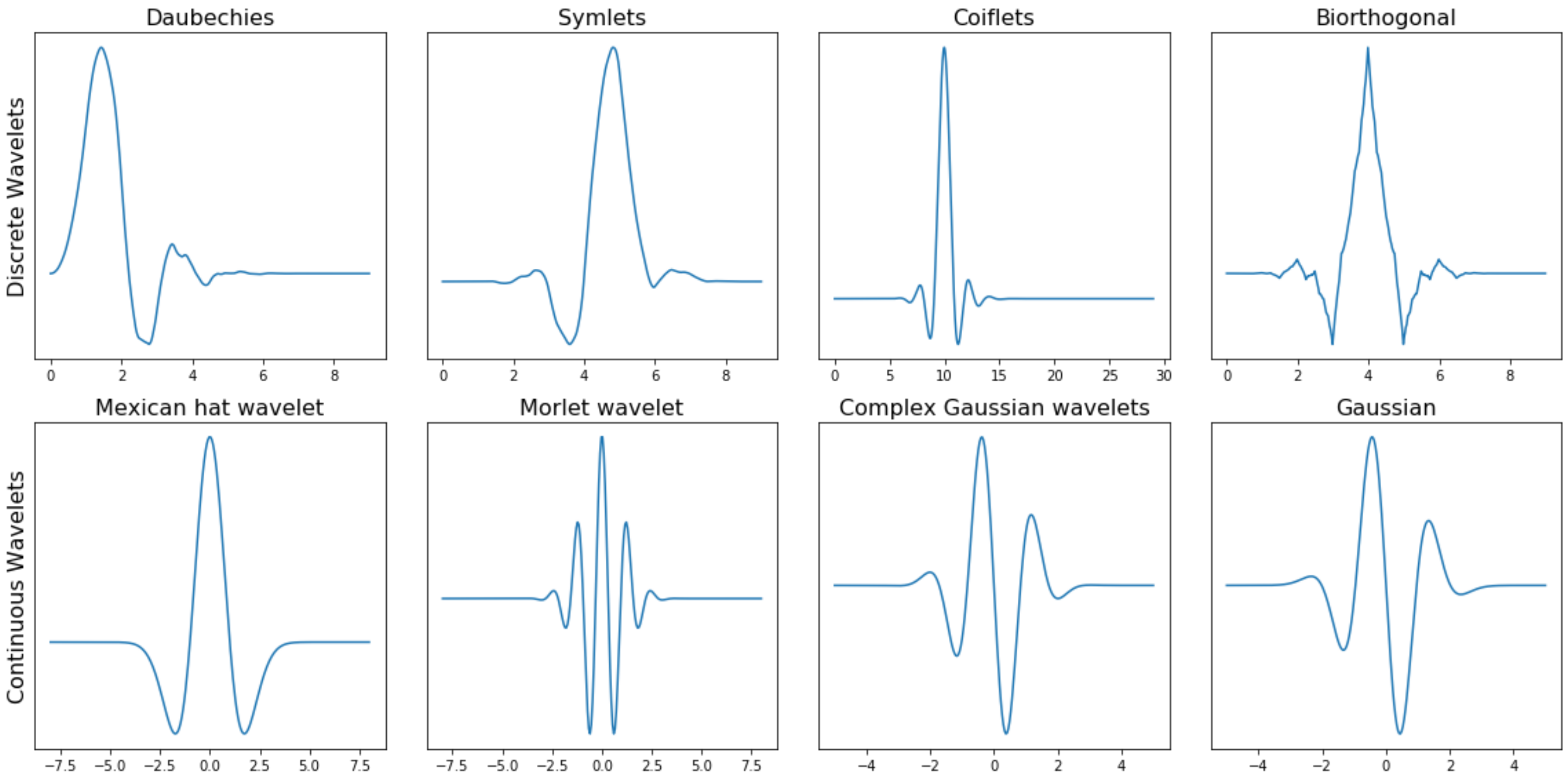
with respect to the mother $\Psi(\cdot)$.

- $k < 1$ Compresses the signal
- $k > 1$ Dilates the signal

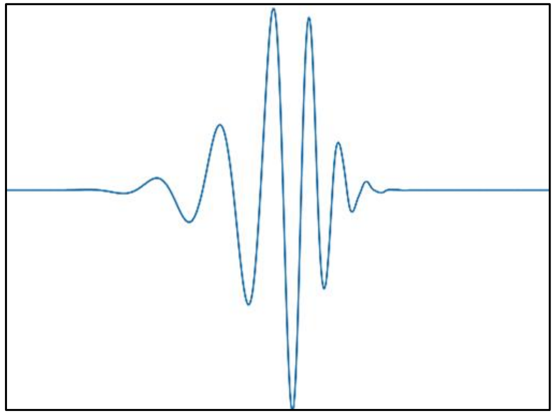
A wavelet browser to play with
http://wavelets.pybytes.com/

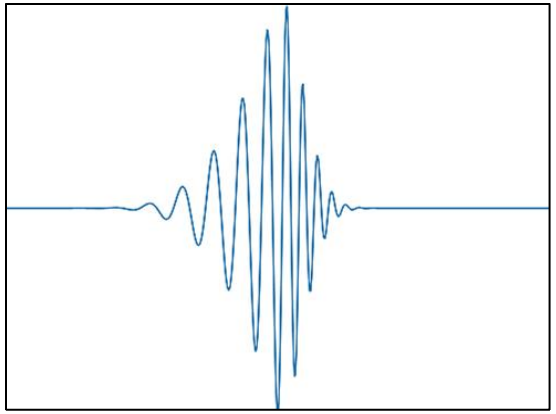
A (partial) wavelet dictionary

Many different possible choices for the mother wavelet function



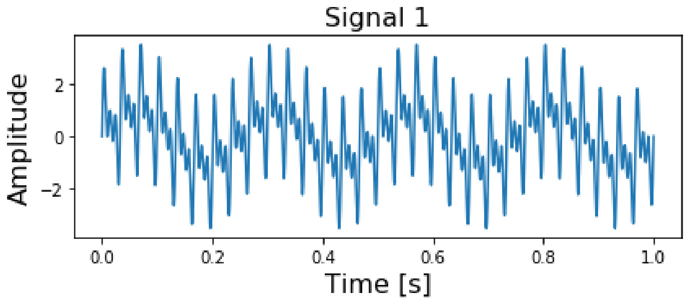
Scaling/dilation is akin to (sort of) frequency





High scale

- Stretched wavelet
- Slowly changing, coarse features
- Low frequency



Low scale

- Compressed wavelet
- Rapidly changing details
- High frequency

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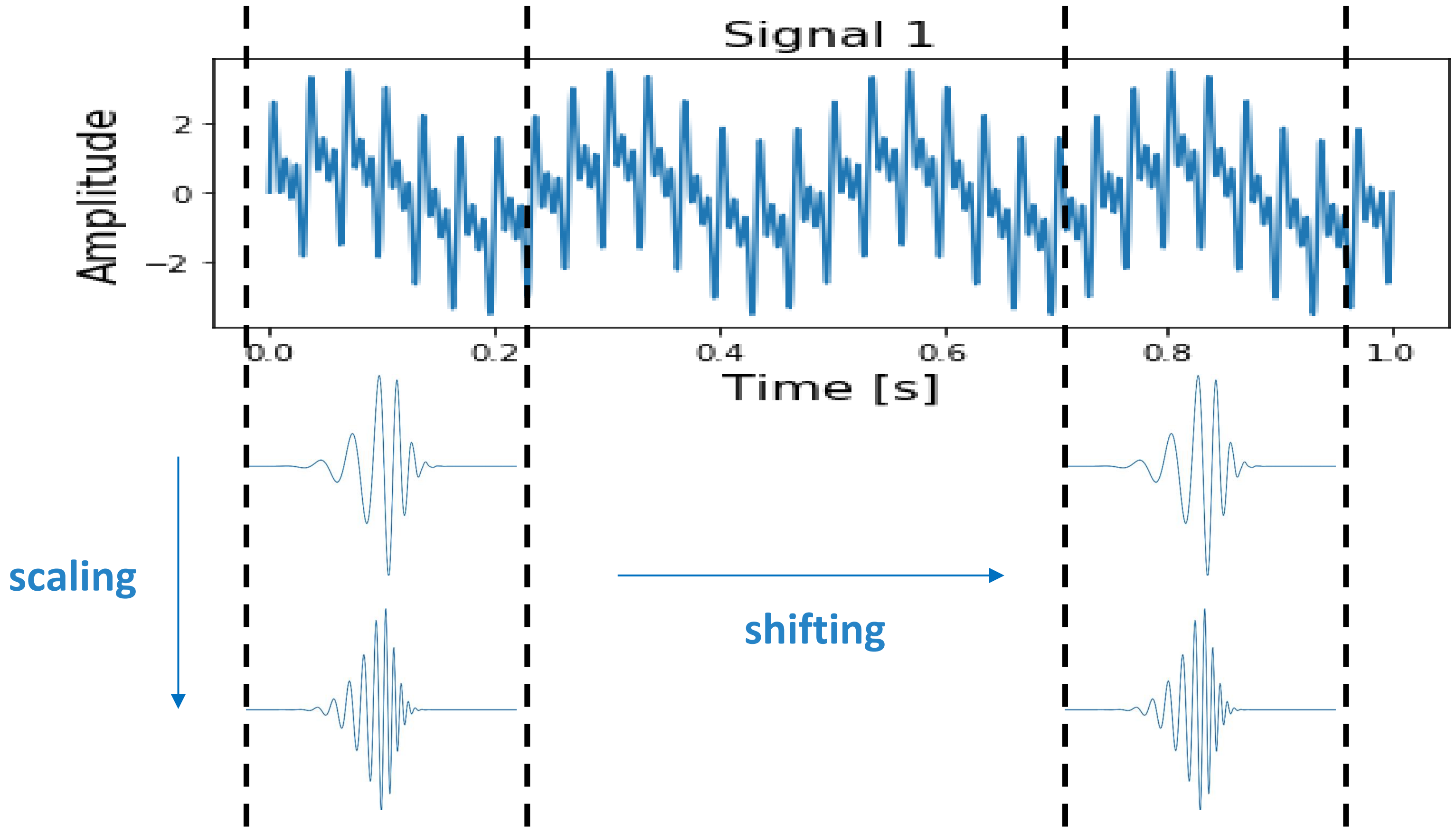
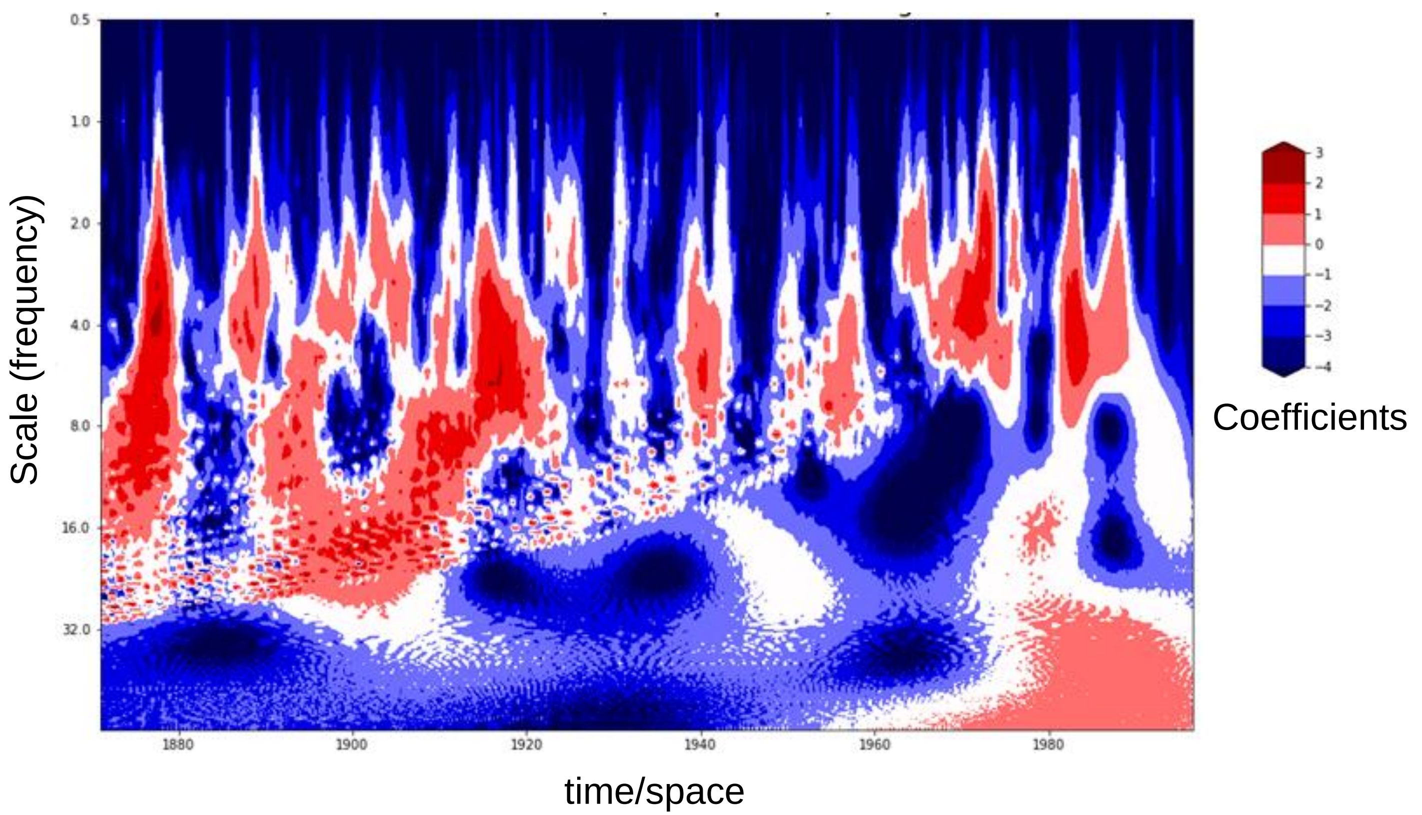
Shifting moves the wavelet in time/space on the signal



Compute coefficients of signal-wavelet match across all scales and shifts

Coefficient Plot



This is once-again the power-spectrum of the signal, revealed by the (continuous) Wavelet Transform

Discrete Wavelet Transform (DWT)

- Use a finite set of scales and shifts rather than “any possible value” as in the continuous wavelet transform
- Subset scale continuous values using power-of-two values with step 1 (and translates proportionally to scale if decimated)
- Key aspects
  - Efficient and sparse representation
  - Orthnormal basis
  - Can always be inverted

Using the WT in PR Applications (I)

Human Activity Recognition Using Smartphones Dataset (Reyes-Ortiz et al, 2012)

9-components sensor measurements of people doing different activities (walking, laying, standing, …)

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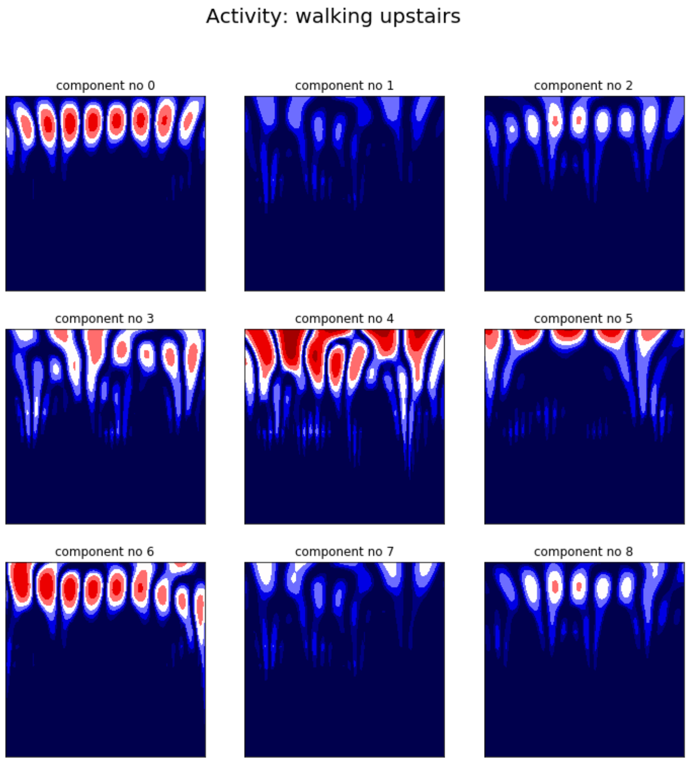
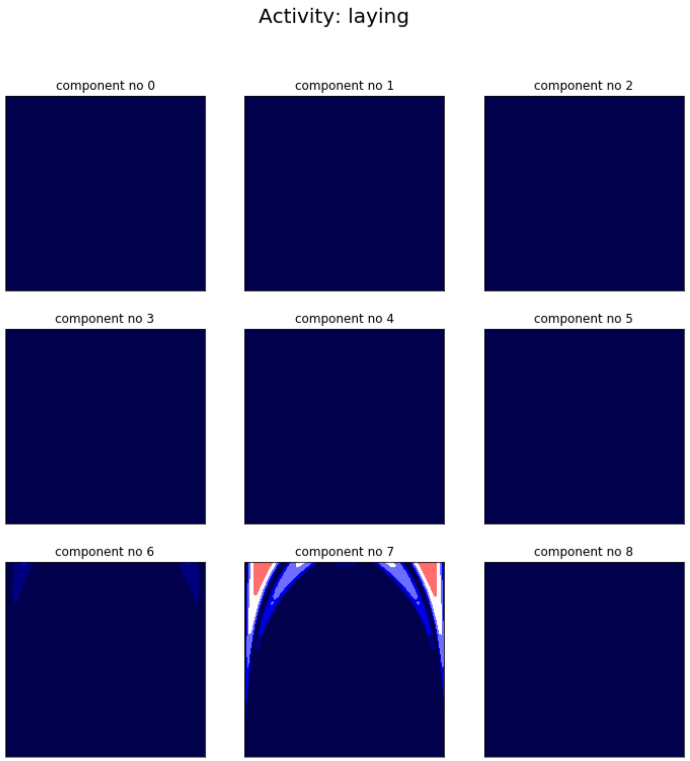
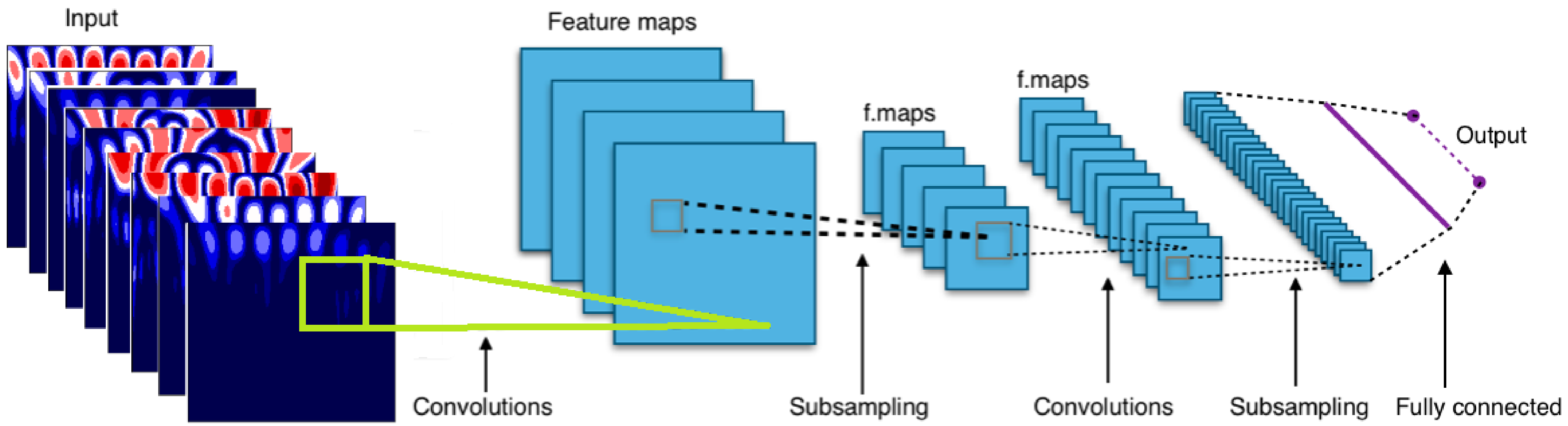
Using the WT in PR Applications (II)





Spectrograms for different activities alone lead to a classification accuracy of 0.91

Using the WT in PR Applications (III)



Using wavelet transform + convolutional neural networks => 0.96

Code

- PyWavelets - Wavelet transforms in Python
- Wavelet Toolbox – Wavelet transforms in Matlab

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Take Home Messages

- Fourier transform
  - Basis functions: sinusoids
  - Only offers frequency information
- Wavelet transforms
  - Basis functions: small waves (wavelets)
  - Frequency and temporal/spatial information
- Wavelets can be more effective on discontinuous and bursty data

Next Lecture

Generative and Graphical Models

- Introduction to a module of 15 lectures
- A refresher on probabilities
  - Probability theory
  - Conditional independence
  - Inference and learning in generative models
- Graphical models representation
- Directed, undirected and dynamic graphical models