Numerical Methods and Optimization (year 2016/17)

The program below is a preliminary schedule. For the actual content of the lectures, we refer to the log (*Registro delle lezioni*) on `http://unimap.unipi.it/`, which will be updated periodically.

COURSE PROGRAM

- Topology and calculus background
  - Sets in the Euclidean space: open sets, closed sets, bounded sets
  - Sequences and limits in the Euclidean space
  - Functions of several variables: continuity, directional derivatives and differentiability, second order derivatives, first and second order Taylor's formulas
- Linear algebra background
  - Normal, unitary, hermitian, positive definite, reducible matrices
  - Schur canonical form
  - Quadratical forms
  - Matrix norms induced by vector norms
- Convex functions, convex sets and optimization problems
  - Convex functions and convex sets: properties and characterizations
  - Classification of optimization problems: local and global minima
  - Properties of convex optimization problems
- Optimality conditions for unconstrained optimization
  - First order necessary and sufficient optimality conditions
  - Second order necessary and sufficient optimality conditions
- Direct and iterative methods for linear systems
  - Matrix factorizations and elementary matrices
  - LU, QR and LL^H factorizations; Gauss, Householder and Cholesky methods
  - Jacobi and Gauss-Seidel methods

  - Convergence of iterative methods
  - The conjugate gradient method for linear systems
  - Brief outline of other method based on Krylov subspaces and Arnoldi factorization.

- Iterative methods for nonlinear systems
  - General notions and conditions for convergence
  - Newton-Raphson method and its convergence; the case of convex functions

- Solution methods for unconstrained optimization
  - Gradients methods with exact and inexact line search
  - Conjugate gradient methods with exact and inexact line search
  - Newton methods and quasi-Newton methods
  - Free derivatives methods

- The least-squares problem
  - The linear problem and the normal equations
  - Solving the linear problem through QR factorization and SVD decomposition
  - Solving the linear problem through the conjugate gradient method

- Iterative methods for computing eigenvalues
  - Bauer-Fike theorem
  - The power method and orthogonal iteration
  - Unitary reduction to tridiagonal form
  - The QR (Francis) method for eigenvalues and its convergence

- Optimality conditions for constrained optimization
  - Tangent cone and first-order feasible direction cone
  - First order optimality conditions
  - Critical cone
  - Second order optimality conditions

- Lagrangian duality
  - Lagrangian relaxation

  - Lagrangian dual
  - Weak and strong duality theorems
- Solution methods for constrained optimization
  - Frank-Wolfe method
  - Penalty methods
  - Barrier methods
- The fast Fourier transform
  - The Fourier matrix and its properties
  - The algorithm and some applications
- Support Vector Machines (SVMs) for classification problems
  - Linear SVMs
  - Kernel functions
  - Nonlinear SVMs